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Quantifying Information Flow between Two Chaotic Semiconductor Lasers Using Symbolic Transfer Entropy *

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Symbolic transfer entropy (STE) is employed to quantify the dominant direction of information flow between two chaotic-semiconductor-laser time series. The information flow in unidirectionally and bidirectionally coupled systems was analyzed systematically. Numerical results show that the dependence relationship can be revealed if there exists any coupling between two chaotic semiconductor lasers. More importantly, in both unsynchronized and good synchronization regimes, the STE can be used to quantify the direction of information flow between the lasers, although the former case leads to a better identification. The results thus establish STE as an effective tool for quantifying the direction of information flow between chaotic-laser-based systems.

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Chaos synchronization is somewhat counterintuitive but interesting because chaotic signals are highly unpredictable and the dynamics of chaotic systems are sensitive to parameter mismatches. Nevertheless, chaos synchronization provides potential applications in secure communication and spread-spectrum communication, and thus has been widely investigated during the last two decades.[1−8] Besides synchronization, it is also of vital importance to detect the dominant direction of information flow between coupled systems. In fact, there may exist asymmetrical couplings or more than one information channel between the systems, e.g., between two chaotic semiconductor lasers.[3,5] After the dominant information flow is revealed, the mechanisms of directional interactions can be understood. Furthermore, quantifying the direction of information flow may also help us to understand coupling among other dynamical nonlinear systems. For example, in neuroscience, understanding the direction of information flow (possible interactions) in epileptogenic networks could improve the evaluation of epilepsy patients as candidates for resective therapies. All of these considerations motivate investigations to quantify the direction of information flow between lasers.

Several approaches have been developed to reveal and quantify coupling direction: state-space-based methods,[9,10] methods based on the concept of Granger causality,[11,12] and information-theoretic techniques,[13,14] etc. The latter group based on information theory makes no assumptions on the system model, and thus deserves more attention. Among the group, transfer entropy (TE)[15] proposed by Schreiber has been proven to be an alternative measurement, which can infer effectively causal relationships. Interestingly, a technique of symbolization has been adopted to improve the estimation of TE, which is called the symbolic transfer entropy (STE).[16,17] The advantages of STE are its robustness and extremely fast computation speed. For this reason, we here adopt STE to quantify the direction of information flow between chaotic laser diodes.

In this Letter, we show, for the first time, how to effectively detect the direction of information flow between two chaotic semiconductor lasers. To this end, we first test the STE on a master-slave configuration in open-loop and closed-loop schemes,[4] proving that the results make sense for quantifying information flow. Then we apply it to a bidirectional scheme.[3,5] Additionally, our results could throw light on the possibility of inferring the direction of information transfer from the experimental time series.

Our rate equation model consisted of the complex slowing varying amplitude of the electric field $E(t)$ and the carrier number inside the cavity $N(t)$ for each laser and reads as follows:[3,5]

$$\begin{aligned}\frac{dE_{1,2}(t)}{dt} =& \frac{1}{2}(1+i\alpha)[G_{1,2}(t)-1/\tau_p]E_{1,2}(t) \\ &+ k_{1,2}E_{1,2}(t-\tau)\exp[-i2\pi f_{1,2}\tau] \\ &+ \sigma_{1,2}E_{2,1}(t-\tau_c)\exp[-i(2\pi f_{2,1}\tau_c \\ &\pm 2\pi\Delta f t)], \end{aligned} \quad (1)$$

$$\frac{dN_{1,2}(t)}{dt} = \frac{I_{1,2}}{e} - \frac{N_{1,2}(t)}{\tau_n} - G_{1,2}(t)|E_{1,2}(t)|^2. \quad (2)$$

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Note that f_1 and f_2 are the optical frequency of the solitary lasers, and $\Delta f = f_1 - f_2$ is the frequency detuning between the lasers. Their device parameters are assumed to be identical: $\alpha = 5$ is the linewidth enhancement factor, $G_{1,2}(t) = g(N_{1,2}(t) - N_0)/(1 + s|E_{1,2}(t)|^2)$ is the optical gain (where $g = 1.5\times10^4\,\mathrm{s}^{-1}$ is the differential gain, $N_0 = 1.5\times10^8$ is the carrier number at transparency, $s = 5\times10^{-7}$ is the gain saturation coefficient), $\tau_p = 2$ ps is the photon lifetime and $\tau_n = 2$ ns is the carrier lifetime, $e = 1.602\times10^{-19}$ C is the electronic charge. In this work the bias current has been fixed to $1.5I_{\mathrm{th}}(I_{\mathrm{th}} = 14.7$ mA) for both lasers. For unidirectional coupling, the feedback strengths and the coupling strengths are specified, $k_1 = 20\ \mathrm{ns}^{-1}$, $k_2 = 0\ \mathrm{ns}^{-1}$ (open-loop in Figs. 1 and 2) or $k_2 = 20\ \mathrm{ns}^{-1}$ (closed-loop), $\sigma_2$is varied and$\sigma_1 = 0\ \mathrm{ns}^{-1}$; for bidirectional coupling, there is no self-feed-back loop, i.e., $k_1 = k_2 = 0\ \mathrm{ns}^{-1}$, $\sigma_1(\sigma_2)$ is fixed at $25\ \mathrm{ns}^{-1}$ and the other is varied. The feedback and coupling delay times are selected as $\tau = \tau_c = 1$ ns. It is worth mentioning that with these parameters, the lasers operate under chaotic regimes, where more than one positive Lyapunov exponent exists.[18]

The synchronization quality is measured by cross correlation coefficient C in both the unidirectional and the bidirectional coupling systems. The definition is[4]

$$C = \frac{\langle[I_1(t+\Delta t_s) - \langle I_1\rangle][I_2(t) - \langle I_2(t)\rangle]\rangle}{\sqrt{\langle[I_1(t) - \langle I_1\rangle]^2\rangle\langle[I_2(t) - \langle I_2(t)\rangle]^2\rangle}}, \tag{3}$$

where $I(t) = |E(t)|^2$ is the output intensity, $\langle\cdot\rangle$ stands for the time average, and Δt_s is the time shift. It should be noted that different types of synchronization can be found in coupled chaotic systems. However, when the coupled systems are completely synchronized, it may be difficult to identify the direction of information flow. Moreover, due to the influence of noises and other factors (e.g. parameter mismatch), one cannot easily obtain complete synchronization. Nevertheless, note that the phenomenon of complete synchronization exists in practical applications and its stability has been investigated in the literature. Thus, in this study we focus on isochronous or generalized synchronization for unidirectional case, and on leader-laggard synchronization for bidirectional case. For unidirectional coupling, $\Delta t_s = -\tau_c$; while for bidirectional coupling, $\Delta t_s = \pm\tau_c$ since the leader-laggard relationship is changed by adjusting the two bidirectional coupling strengths.

To construct the STE measurement, we refer to the original STE[16] and the improvement of STE.[19] Here we demonstrate that the quantifier allows us to determine the dominant direction of information flow between two SLs. As we know, the STE is based on the Bandt and Pompe (BP) symbolization technique,[20] therefore the procedure of STE is as follows: (i) let us generate an ordinal pattern as $X_i \mapsto (x_{i-(D_x-1)\tau_e}, x_{i-(D_x-2)\tau_e}, \ldots, x_{i-\tau_e}, x_i)$, where $D_x \in \mathrm{N}^+(\tau_e \in \mathrm{N}^+)$ is the embedding dimension (delay), when a time series $\{x_t, t = 1,\ldots,N\}$ is given. (ii) To each time i we assign a D_x-dimensional vector which results from the evaluation of the time series at times $i-(D_x-1)\tau_e,\ldots,i-\tau_e,i$. (iii) By the ordinal pattern of D_x related to the time i, one can define $x_{i-r_{D_x-1}\tau_e} \le x_{i-r_{D_x-2}\tau_e} \le \cdots \le x_{i-r_1\tau_e} \le x_{i-r_0\tau_e}$, then a unique permutation$\hat{x}_i = (r_0, r_1, \ldots, r_{D_x-1})$ of $(0,1,\ldots,D_x-1)$ is obtained. (iv) In this way, any vector X_i is converted into a unique symbol $\hat{x}_i$. For all the $D_x!$ possible permutations, the associated relative frequencies are defined by

$$p(\hat{x}_i) = \frac{\#\{i|1+(D_x-1)\tau_e \le i \le N, i \text{ has type } \hat{x}_i\}}{N-(D_x-1)\tau_e}, \tag{4}$$

where # is the "number". Obviously, the same procedure can be carried out for time series $\{y_t, t = 1,\ldots,N\}$. For detailed descriptions of the BP method, one can refer to Ref. [20], where some examples are presented. (v) According to permutation probability distributions, the STE is naturally computed by

$$T^S_{Y,X} = \sum p(\hat{x}_{i+\delta}, \hat{x}_i, \hat{y}_i)\log\frac{p(\hat{x}_{i+\delta}|\hat{x}_i,\hat{y}_i)}{p(\hat{x}_{i+\delta}|\hat{x}_i)}. \tag{5}$$

Note that the sum runs over all symbols and δ stands for a time step, and $T^S_{X,Y}$ is defined in complete analogy. Finally, a directionality index (ΔT) is obtained, $\Delta T = T^S_{X,Y} - T^S_{Y,X}$, which allows us to detect the dominant direction of information flow between systems X and Y. Definitely speaking, if $\Delta T > 0$, $X(Y)$ is the driving (responding) system and vice versa.

The intensity time series are numerically calculated by integrating Eqs. (1) and (2) with time steps of $\Delta t = 0.1$ ps, where the length of the time series includes $N = 10^4$ data points and the sampling period is $\Omega_s = 1$ ps. Following Ref. [16], we restrict the control parameters of STE as follows: embedding dimension $[4 \le D_x(D_y) \le 7]$, embedding delay $(\tau_e = 10)$, time step $(\delta = 1)$. Note that X denotes one semiconductor laser (SL1) and Y the other (SL2) hereafter.

To explore the ability of the STE measurement to quantify the direction of information flow, the ΔT for a master-slave configuration is calculated, where the open-loop and closed-loop schemes are both included. First, the frequency detuning $\Delta f = 0$ and the embedding dimensions $D_x = D_y = 7$ are specified. As shown in Fig. 1, in both the schemes, for very low coupling strength, the outputs of SL1 and SL2 are unsynchronized, yielding a larger ΔT. This indicates that SL2 receives a large amount of information from SL1. With the increment of coupling strength, ΔT decreases, indicating good synchronization between the lasers. As expected, when SL1 and SL2 achieve perfect synchronization, they output near-identical dynamics

and therefore the dominant role of SL1 weakens. However, SL1 still influences SL2 since ΔT never reaches zero.

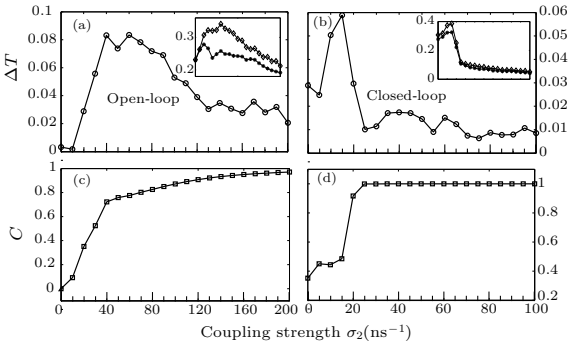


Fig. 1. Results for the unidirectional coupling system: ΔT (first row), isochronous correlation plot (second row) for varying coupling strength. Open-loop (left column) and closed-loop (right column) are included. The inset shows the evolutions of $T^S_{X,Y}$ (with diamonds) and $T^S_{Y,X}$ (with asterisks) on σ_2.

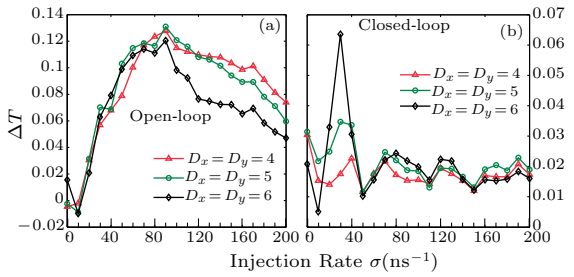


Fig. 2. (Color online) The evolution of ΔT as a function of injection rate for different but symmetrical embedding dimensions.

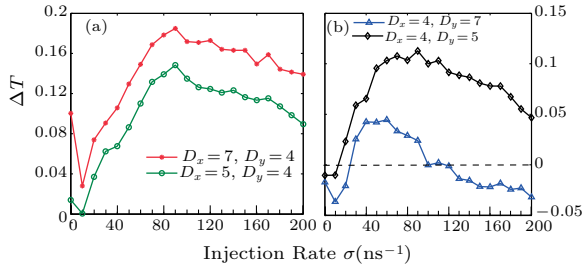


Fig. 3. (Color online) The evolution of ΔT as a function of injection rate for different and asymmetrical embedding dimensions in an open-loop scheme.

To gain insight into the robustness and feasibility of the STE measure, we perform numerical simulations for different but symmetrical embedding dimensions. Figure 2 presents the results for open-loop and closed-loop schemes: $D_x = D_y = 4$ (red solid line with triangles), $D_x = D_y = 5$ (green solid line with circles), and $D_x = D_y = 6$ (black solid line with diamonds). The results are consistent and robust for varying the embedding dimension provided that the symmetry is maintained for the time series of both lasers ($D_x = D_y$).

The influence of an asymmetrical selection of embedding dimensions on the inferring direction of information flow between unidirectionally coupled chaotic laser diodes is further investigated. For the sake of concision, only the results for an open-loop scheme are shown in Fig. 3, where $D_x > D_y$ (a) and $D_x < D_y$(b) are considered. It can be concluded that the STE measurement is sensitive to the selection of embedding dimensions: in our configuration, the direction is correctly detected for $D_x > D_y$, while it can be biased if D_y is much larger than D_x (see $D_x = 4, D_y = 7$). It should be stressed that we have evaluated the results for ΔT for different embedding delays, which show no significant impact on the estimate of ΔT in unidirectionally coupled semiconductor lasers. For concision, the corresponding results are not presented here (see Ref. [16] for details).

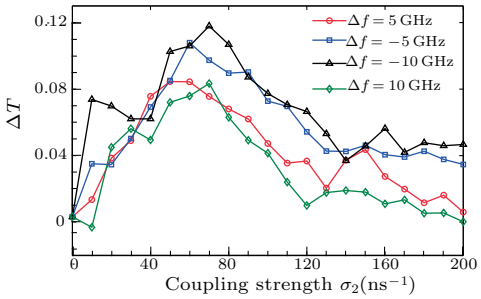


Fig. 4. (Color online) The effect of frequency detuning in an open-loop scheme.

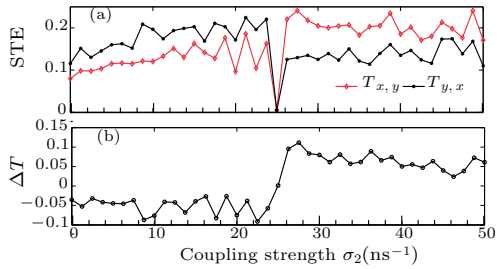


Fig. 5. The results for a bidirectional coupling system: (first row) the evolutions of $T^S_{X,Y}$ (diamonds) and $T^S_{Y,X}$ (asterisks), ΔT (second row) on σ_2. The σ_1 is fixed at 25 ns^{-1}.

The effects of frequency detuning are discussed next. Figure 4 shows the results for positive and negative detuning in an open-loop scheme. It can be seen that frequency detuning seems not to affect the information flow between the lasers qualitatively. As we know, the detuning only degrades the synchronization degree, however it will not change the leader-laggard role in unidirectional coupling. As a result, only a slight change of ΔT can be found when Δf is varied from -10 GHz to 10 GHz. Moreover, the results for a closed-loop scheme are completely consistent (not shown in the figure). In this sense, the computation of ΔT can reveal effectively the direction of information flow between the lasers.

In addition, quantifying bidirectional interactions

between two lasers may be more interesting. The leader-laggard chaos synchronization is achieved when the two lasers are coupled together with extremely unsymmetrical bidirectional coupling strengths. To this end, we first fix $\sigma_1 = 25$ ns^{-1} and take different values of σ_2, and the second vice versa. All the other parameters are identical for the lasers. Also, $D_x = D_y = 7$ was assumed. Figure 5 shows that ΔT detects the correct unsymmetrical bidirectional coupling. Specifically, for $\sigma_1 > \sigma_2$, we obtain $\Delta T < 0$, then SL2 is the leader, and vice versa. Note that when $\sigma_1 = \sigma_2 = 25$ ns^{-1} the two lasers operate symmetrically, therefore the value of ΔT is close to zero. Additionally, the results for fixing σ_2 and sweeping σ_1 are completely analogous and not shown in the figure.

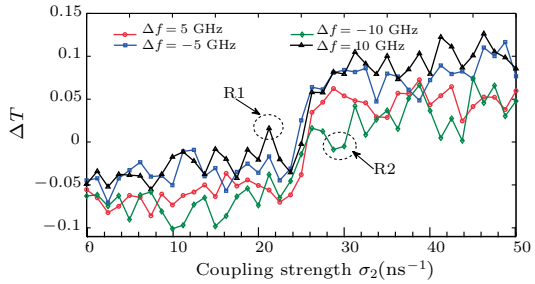


Fig. 6. (Color online) The effect of frequency detuning in bidirectional coupled lasers.

In Fig. 6, we present the results for the influence of the detuning. It has been found that the leader-laggard relationship is determined by the combination impact of the frequency detuning and the coupling strengths, when the detuning is increased (decreased) to a critical value. For the sake of clarity, we focus on the results for $\Delta f = -10$ GHz and $\Delta f = 10$ GHz. One can deduce that a slightly large detuning may change the relationships for information flow between SL1 and SL2. For example, in region $R1$ (marked with an ellipse), as shown in Fig. 6, the dominant direction of information flow is from SL1 to SL2 since ΔT becomes positive, which is contrary to that of the no detuning case [Fig. 5 (second row)]. Similar results can be found in region R2 ($\Delta T < 0$), where SL2 becomes the leader again. Comparing the results for $\Delta f = -10$ GHz to those for $\Delta f = 10$ GHz, one is in a position to deduce that the influence of the lower frequency laser on the higher one is stronger than that in the opposite case, which coincides with Ref. [21]. Based on these results, we can definitely say that the computation of the ΔT is able to quantify the details of the direction of information flow in the bidirectional system.

In conclusion, we have demonstrated numerically that STE can be applied to quantify the dominant direction of information flow between two chaotic semiconductor lasers. The results for STE are robust with respect to the selection of the embedding dimension in unidirectionally and bidirectionally coupled lasers when the symmetrical condition ($D_x = D_y$) is maintained. To further show the feasibility of the STE, a quantitative analysis of noise robustness and a generalization of the method to other semiconductor laser based systems will be our future focuses.

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