CHIN. PHYS. LETT. Vol. 29, No. 3 (2012) 030306

Pulse Designed Coherent Dynamics of a Quantum Dot Charge Qubit *

CAO Gang(曹刚), WANG Li(王力), TU Tao(涂涛)**, LI Hai-Ou(李海鸥),
XIAO Ming(肖明), GUO Guo-Ping(郭国平)**

Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026

(Received 20 October 2011)

We propose an effective method to design the working parameters of a pulse-driven charge qubit implemented with double quantum dot. It is shown that intrinsic qubit population leakage to undesired states in the control and measurement process can be determined by the simulation of coherent dynamics of the qubit and minimized by choosing proper working parameters such as pulse shape. The result demonstrated here bodes well for future quantum gate operations and quantum computing applications.

PACS: 03.67.Lx, 03.65.Yz, 03.65.Ta DOI: 10.1088/0256-307X/29/3/030306

A central issue in quantum computation is coherence, i.e., qubits must survive in high degree quantum coherence states for a long time without decay. During the past decade, significant progress has been made on solid state quantum computing based on semiconductor quantum dots. Quantum dot qubits are of particular interest because of their advantages of large-scale integration and flexibility in control by conventional electronic circuits. Semiconducting qubits utilize charge[1−3] or spin degrees of freedom,[4−9] or hybrid quantization states in nanowire quantum dot with strong spin-orbit coupling.[10] In the present study, we focus on charge qubit. The qubit states correspond to different charge states in a double quantum dot (DQD) that can be manipulated by means of external electrical pulses and then measured by the current through the system[1] or a properly coupled quantum point contact detector.[2]

However, the charge qubits demonstrated in experiments so far all have relatively short coherence time T_2 no more than 1 ns and high probability of gate errors.[1,11] One of the causes of these problems is extrinsic interaction between the environment and qubits resulting in decoherence. Specifically, the qubits are strongly influenced by low-frequency fluctuation, cotunneling process, and acoustic phonons. The cotunneling and electron-phonon interaction can be strongly reduced at low temperatures. An insulated electrostatic electrode can be used to significantly suppress background charge noise in quantum dot devices. Thus the only important decoherence channel left is $1/f$ noise due to trapping of electrons from the impurity centers.[12] A new decoherence channel could open up when qubits are manipulated or measured. The leakage of a gate operation could arise from electron transition to undesired states due to the typical multilevel structures of charge qubit in a DQD. This intrinsic gate error is crucial since it determines the ultimate performance of the quantum gates and cannot be eliminated by reducing the environmentally caused decoherence.

In this Letter, we propose a simple method to determine the leakage of pulse-driven quantum manipulations and choose appropriate working parameters in a system of semiconducting DQD.[1] Our method is based on a simulation of coherent dynamics of the qubit and the leakage can be characterized quantitatively by summing up the electron transition probabilities to all undesired states of a multilevel charge qubit. It depends strongly on energy level structure and transition matrix elements. The former is determined by device parameters and can be optimized in a sample design. The latter is related to external control parameters which we call the working parameters for simplicity. For instance, external gate pulse shape is a working parameter. For a given qubit, the gate leakage is sensitive to their working parameters and thus can be minimized by the use of appropriate working parameters. Here we consider the case of different pulse shapes to compare many types of intrinsic gate errors resulting from qubit population leakage to undesired states in the control and measurement process. The time evolution properties of the qubit can be improved and gate error is minimized by optimizing the working parameters of the qubit. The method is general and can also be applied to a universal gate implemented with solid state (semiconductor or superconductor) charge qubits.[1,13]

Model and method: A DQD provides a simple basic element for qubit. We consider one-electron states $|L\rangle$ and $|R\rangle$ in the DQD, in which an electron occupies the left dot or the right dot, respectively. When the interdot tunneling is allowed by applying the external gate pulse, the electron state can be expressed

*Supported by the National Basic Research Program of China under Grant No 2011CBA00200, and the National Natural Science Foundation of China under Grant Nos 11074243 and 10934006.

**To whom correspondence should be addressed. Email: gpguo@ustc.edu.cn; tutao@ustc.edu.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

with a linear superposition of $|L\rangle$ and $|R\rangle$ and the time evolution of the system is well described by the coherent oscillations between states $|L\rangle$ and $|R\rangle$. This system is referred to a charge qubit as a building block of a quantum computer. However the above approximation is not enough for a realistic modeling of the system. The empty state $|0\rangle$, in which there is no electron in the DQD, and the doubly occupied state $|2\rangle$, in which two electrons occupy different dots, are also taken into account. In the present study, we do not consider the spin freedom.

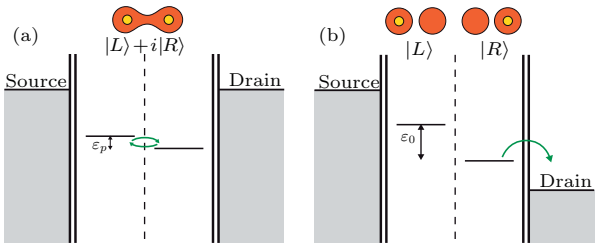

Fig. 1. Schematic illustration of manipulation (a) and measurement (b) processes of a charge qubit in the double quantum dot and the source and drain leads. (a) Coherent oscillations appear between the states $|L\rangle$ and $|R\rangle$ for an applied pulse detuning ϵ_p in the manipulation process. (b) The tunneling current is observed for $|R\rangle$ state in the measurement process, while it is absent for the $|L\rangle$ state.

The total Hamiltonian of the system contains terms describing the electron reservoirs, DQD and the tunnel coupling between them, $H = H_{\rm res} + H_{\rm DQD} + H_{\rm tun}$. The reserviors read as noninteracting electrons

$$H_{\rm res} = \sum_s E_s a_s^\dagger a_s + \sum_d E_d a_d^\dagger a_d,$$

where E_s and E_d denote the energy levels in the electrodes and $a_s^\dagger$, a_s, $a_d^\dagger$, a_d are creation and annihilation operators for an electron in the source and drain leads, respectively. The DQD is described usually by

$$H_{\rm DQD} = E_{\rm L} a_{\rm L}^\dagger a_{\rm L} + E_{\rm R} a_{\rm L}^\dagger a_{\rm L} + \Omega_0 (a_{\rm L}^\dagger a_{\rm R} + a_{\rm R}^\dagger a_{\rm L}),$$

where the subscripts L and R denote correspondingly the electron state in the left and right dots, Ω_0 is the coupling between them. The electron tunneling between the DQD and the leads is given by

$$H_{\rm tun} = \sum_s \Omega_s (a_s^\dagger a_{\rm L} + a_{\rm L}^\dagger a_s) + \sum_d \Omega_d (a_d^\dagger a_{\rm R} + a_{\rm R}^\dagger a_d).$$

In order to obtain an equation of motion for the density operator σ of the system, we employ the time-dependent Schröinger equation technique.[14] The many-body wavefunction describing the entire system can be written as

$$|\Psi(t)\rangle = \Big[b_0(t) + \sum_s b_{{\rm L},s}(t) a_{\rm L}^\dagger a_s + \sum_{s,d} b_{s,d}(t) a_d^\dagger a_s + + \sum_s b_{{\rm R},s}(t) a_{\rm R}^\dagger a_s + \cdots \Big] |0\rangle,$$

where $b(t)$ are the time-dependent probability amplitudes to find the system in the corresponding states. The quantum evolution of the whole system is determined by the Schröinger equation $i\hbar \frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$ and the corresponding density matrix is given by $\sigma = |\Psi\rangle\langle\Psi|$.

Control and measurement process: For coherent manipulation, we consider applying a nonadiabatic pulse with maximum detuning $\epsilon_p = E_{\rm L} - E_{\rm R} \approx 0$ and width t_p to the DQD as shown in Fig. 1(a). The system is in the Coulomb blockade regime, where direct tunneling into and out of the DQD is blocked. The coherent oscillation between the states $|L\rangle$ and $|R\rangle$ is expected. Notice that there are also transitions between the one-electron states $|L\rangle$ and $|R\rangle$ and undesired states such as the empty state $|0\rangle$ and the doubly occupied state $|2\rangle$. In the corresponding Fock space $\{|L\rangle, |R\rangle, |0\rangle, |2\rangle\}$, the rate equations for the density matrix of the system are given by

$$\begin{aligned}
\frac{d\sigma_{\rm LL}}{dt} &= 2\Gamma_{\rm L}\sigma_{00} + 2\Gamma_{\rm R}\sigma_{22} + i\Omega_0(\sigma_{\rm LR} - \sigma_{\rm RL}),\\
\frac{d\sigma_{\rm RR}}{dt} &= 2\Gamma_{\rm R}\sigma_{00} + 2\Gamma_{\rm L}\sigma_{22} - i\Omega_0(\sigma_{\rm LR} - \sigma_{\rm RL}),\\
\frac{d\sigma_{\rm LR}}{dt} &= i\epsilon_p\sigma_{\rm LR} + i\Omega_0(\sigma_{\rm LL} - \sigma_{\rm RR}),\\
\frac{d\sigma_{\rm RL}}{dt} &= -i\epsilon_p\sigma_{\rm RL} - i\Omega_0(\sigma_{\rm LL} - \sigma_{\rm RR}),\\
\frac{d\sigma_{00}}{dt} &= -2(\Gamma_{\rm L} + \Gamma_{\rm R})\sigma_{00},\\
\frac{d\sigma_{22}}{dt} &= -(\Gamma_{\rm L} + \Gamma_{\rm R})\sigma_{22},
\end{aligned} \tag{1}$$

where $\Gamma_{\rm L}$ and $\Gamma_{\rm R}$ is obtained by summing up the electron tunnel coupling to the reservoirs.

Next we consider the measurement process where the DQD is adjusted to large positive detuning ϵ_0 following the pulse. If the coherent oscillation ends up in $|R\rangle$ after the control process, the electron can tunnel out to the drain electrode and contribute to the current, while no current is expected for $|L\rangle$. Thus the charge state of the qubit after the manipulation is read out using the current through the system. In this regime, the rate equations are obtained as

$$\begin{aligned}
\frac{d\sigma_{\rm LL}}{dt} &= 2\Gamma_{\rm L}\sigma_{00} + 2\Gamma_{\rm R}\sigma_{22} + i\Omega_0(\sigma_{\rm LR} - \sigma_{\rm RL}),\\
\frac{d\sigma_{\rm RR}}{dt} &= 2(\Gamma_{\rm R} + \Gamma_{\rm L})\sigma_{\rm RR} - i\Omega_0(\sigma_{\rm LR} - \sigma_{\rm RL}),\\
\frac{d\sigma_{\rm LR}}{dt} &= -\frac{1}{2}(\Gamma_{\rm L} + \Gamma_{\rm R})\sigma_{\rm LR} + i\epsilon_0\sigma_{\rm LR} + i\Omega_0(\sigma_{\rm LL} - \sigma_{\rm RR}),\\
\frac{d\sigma_{\rm RL}}{dt} &= -\frac{1}{2}(\Gamma_{\rm L} + \Gamma_{\rm R})\sigma_{\rm LR} - i\epsilon_0\sigma_{\rm RL} - i\Omega_0(\sigma_{\rm LL} - \sigma_{\rm RR}),\\
\frac{d\sigma_{00}}{dt} &= -2\Gamma_{\rm L}\sigma_{00} + \Gamma_{\rm R}\sigma_{\rm RR},\\
\frac{d\sigma_{22}}{dt} &= -\Gamma_{\rm R}\sigma_{22} + 2\Gamma_{\rm L}\sigma_{\rm RR}.
\end{aligned} \tag{2}$$

After solving Eq. (2), we can obtain the time-

dependent current $I_\Psi(t) = e\Gamma_{\rm RR}(\sigma_{\rm RR} + \sigma_{22})$, which depends on the probability of the $|R\rangle$ state. The total tunneling current during the measurement process is obtained by subtracting the static current I_{st} of the steady state $\frac{d\sigma}{dt} = 0$,

$$I = \int [I_\Psi(t) - I_{st}]dt. \tag{3}$$

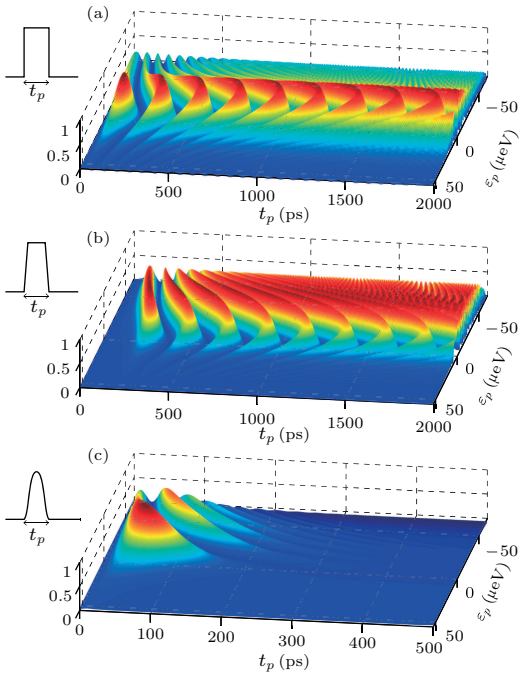

Fig. 2. Three-dimensional plot of pulse-induced current as a function of detuning ϵ and pulse length t_p. The current transport through the device $\langle N\rangle = I/e$ after a single-pulse operation depending on the averaged probability at $|R\rangle$ state is calculated by numerically solving a time-dependent Schröinger equation, demonstrating the coherent qubit dynamics for different applying pulse shapes: (a) rectangular, (b) trapezoidal, (c) Gaussian, respectively.

Coherent dynamics for different pulses: One can simulate the qubit dynamics by solving Eqs. (1) and (2) in series. As shown in Fig. 2, we calculated the expected value of tunneling electrons $\langle N\rangle = I/e$ after the time evolution under various pulse conditions. First, we simulated the qubit evolution under the influence of perfect rectangular pulses as a function of pulse operation time t_p and pulse detuning ϵ_p. Around $\epsilon_p = 0$ we observe strong enhanced oscillations. Second, we applied a trapezoidal pulse with a rise time, a flat top at detuning ϵ_p for a variable time, and a fall time back to the resting point. The trapezoidal pulse was chosen as a reasonable approximation, which includes an electronics-limited, 100 ps rise and fall time. The time axis in Fig. 2(b) includes both the rise and fall times. As presented in Fig. 2(b), the simulation demonstrated asymmetry pattern about $\epsilon_p = 0$, where coherent oscillations are strongly suppressed at positive detuning due to the finite rise time of the pulses. Third, we used a Gaussian through the simulation as shown in Fig. 2(c). In a realistic experiment, filtering a rectangular pulse or accounting for charge noise[15] induces the rise and fall times. At long rise and fall times, pulses appear Gaussian not rectangular. We observed that the oscillation visibility is sensitive to the Gaussian shaping. Oscillations located near $\epsilon_p = 0$ and short times have higher contrast, while they have a significant reduction and to be washed out at other detunings and long times. The simulation results suggested it is due to an increase in the adiabaticity of the pulses.

The shape of the applied pulse strongly effects the amplitude of the state transfer between qubit ($|L\rangle$ and $|R\rangle$ states) and undesired states such as $|0\rangle$ and $|2\rangle$ states. Thus, simulations show that when the rise and fall time are set for different pulse shapes, an obvious oscillation pattern change in the response is observed. Using three typical kinds of pulses as described above, however, the different pronounced coherent oscillation emerge, as in Fig. 2. All of this suggests that working parameters such as pulse shape can be designed for the coherent evolution dynamics of a qubit to further understand the experiment data.[1,2,15]

In conclusion, motivated by the relevance of double quantum dot for realizing a solid state charge qubit, here we perform the simulation to probe pulse controlled coherent dynamics of the qubit. For quantum computation to be a reality, the effects of interactions of qubits with their environment or intrinsic process must be minimized. The validity and efficiency of our approach are demonstrated by applying it to design and optimize the working parameters such as pulse profiles for high-fidelity one qubit coherent dynamics in the presence of intrinsic leakage. Besides a semiconducting double quantum dot qubit, there is a superconductor version,[13] also based on the charge degree of freedom. Nevertheless, the result described here can be valid for various types of solid state charge qubits. More importantly, our protocol is quite robust in the improvement of more complicated gate performances which bodes well for future quantum information studies.[16] Demonstration of pulse designed qubit behavior is an important and first step towards realizing such promise.

References

[1] Hayashi T, Fujisawa T, Cheong H D, Jeong Y H and Hirayama Y 2003 *Phys. Rev. Lett.* **91** 226804
[2] Petersson K D, Petta J R, Lu H and Gossard A C 2010 *Phys. Rev. Lett.* **105** 246804
[3] Cao G, Li H O, Tu T, Zhou C, Hao X J, Guo G C and Guo G P 2009 *Chin. Phys. Lett.* **26** 097302
[4] Hanson R, Kouwenhoven L P, Petta J R, Tarucha S and Vandersypen L M K 2007 *Rev. Mod. Phys.* **79** 1217
[5] Zhang H, Guo G P, Tu T and Guo G C 2007 *Phys. Rev.* A **76** 012335

[6] Guo G P, Zhang H, Hu Y, Tu T and Guo G C 2008 *Phys. Rev.* A **78** 020302
[7] Lin Z R, Guo G P, Tu. T, Zhu F Y and Guo G C 2008 *Phys. Rev. Lett.* **101** 230501
[8] Zhu F Y, Tu T, Hao X J, Guo G C and Guo G P 2009 *Chin. Phys. Lett.* **26** 050301
[9] Wang L J, Cao G, Tu T, Li H O, Zhou C, Hao X J, Guo G C and Guo G P 2011 *Chin. Phys. Lett.* **28** 067301
[10] Nadj Perge1 S, Frolov S M, Bakkers E P A M and Kouwenhoven L P 2010 *Nature* **468** 1084
[11] Petta J R, Johnson A C, Hanson M P, Marcus C M and Gossard A C 2004 *Phys. Rev. Lett.* **93** 186802
[12] Buizert C, Koppens F H L, Pioro Ladriere M, Tranitz H P, Vink I T, Tarucha S, Wegscheider W and Vandersypen L M K 2008 *Phys. Rev. Lett.* **101** 226603
[13] Nakamura Y, Pashkin Y A and Tsai J S *Nature* **398** 786
[14] Gurvitz S A 1997 *Phys. Rev.* B **56** 15215
[15] Dovzhenko Y, Stehlik J, Petersson K D, Petta J R, Lu H and Gossard A C 2011 *Phys. Rev.* B **84** 161302
[16] Shinkai G, Hayashi T, Ota T and Fujisawa T 2009 *Phys. Rev. Lett.* **103** 056802