CHIN. PHYS. LETT. Vol. 29, No. 11 (2012) 114214

All-or-Nothing-Type Kochen-Specker Experiment with Thermal Lights *

SONG Xin-Bing(宋新兵)[1,2], WANG Hai-Bo(王海波)[1], XIONG Jun(熊俊)[1], ZHANG Xiang-Dong(张向东)[1,2]**

[1]*Department of Physics, Applied Optics Beijing Area Major Laboratory, Beijing Normal University, Beijing 100875*
[2]*School of Physics, Beijing Institute of Technology, Beijing 100081*

(Received 9 July 2012)

Recently, the Kochen–Specker theorem has been demonstrated and quantum contextuality has been observed experimentally by using special and independent quantum states. On the other hand, recent investigations have shown that a thermal light source can mimic the two-photon entangled source to perform the imaging effects. We perform an all-or-nothing–type Kochen-Specker experiment using the thermal light source and obtain the same results as those using quantum sources. The physical origin for such a phenomenon is analyzed.

PACS: 42.50.Xa, 42.50.Dv, 03.65.Ta DOI: 10.1088/0256-307X/29/11/114214

In recent years there has been a great deal of interest in studying the ghost imaging with a thermal light source.[1–7] Correlated ghost imaging was proposed by Klyshko[8] for exploring two-photon quantum entanglement in spontaneous parametric down-conversion and demonstrated experimentally by Pittman *et al.*[9] Subsequently, researchers showed that a thermal (or quasi-thermal) light source can mimic a two-photon entangled source in performing the ghost imaging. A very interesting feature of the use of incoherent sources is that they can be used to generate images without the use of lenses.[10–13] Lensless imaging is very useful in some applications.

On the other hand, the problem of whether quantum phenomena can be explained by classical models with hidden variables has been discussed for more than half a century.[14] There are two powerful arguments about such a problem. One is the local hidden-variable theories which assume that the predetermined value of an observable does not depend on what other observables are simultaneously measured in a spatially separated region.[15–25] In 1964, Bell derived his inequality to show that the local hidden-variable theories cannot explain the quantum mechanical predictions. Subsequently, many experiments have been performed to test Bell-type inequalities.[15–25]

The other is the noncontextual hidden-variable (NCHV) theories, which assume that the result of a measurement of an observable is predetermined and independent of a suitable (previous or simultaneous) measurement of any other observable.[26] The NCHV theories reveal a basic opinion in hidden variable theories more directly. However, for a long time, there has not been any experimental disproof of NCHV theories because the Kochen–Specker (KS) theorem designs a rather complex formulation to show that the NCHV is not compatible with quantum mechanics.[26,27] The simplified versions for the KS theorem were given until 1990.[27] In recent years, some experimentally testable schemes have been proposed and the experiments have been performed by means of special quantum states such as photons and neutrons.[28–35] Very recently, a state-independent experiment with trapped ions has been completed, and shows that the results cannot be explained in non-contextual terms.[36] All these experiments to test the KS theorem focused on the quantum states (special or general). It is natural to ask what phenomenon will appear if we use the thermal light source instead of the above quantum sources to perform the KS experiment.

In this work we perform an all-or-nothing–type KS experiment using the thermal light source. Our experimental setup shown in Fig. 1 is similar to that in Ref. [32] with the exception that the thermal light source replaces the single photon source. The thermal light source is provided by a rubidium lamp with a radiation light wavelength 780 nm. The photons are detected by single-photon detectors D1–D8 (Perkin-Elmer SPCM-AQR-14). A 10 nm bandwidth interference filter is placed in front of each detector. In the experiment, the classical thermal light in the required polarized state is prepared by a properly rotated polarizing plate (PP) and a polarizing beam splitter (PBS). Here all the PBSs are set to reflect vertical polarization light.

In fact, a similar experiment using a single photon source by the spontaneous parametric down-conversion process has been performed in Ref. [32], in which the polarization and path degrees of a single photon were used to form a two-qubit system that is equivalent to the original proposal in Ref. [27] which uses spin-1/2 particles. In Ref. [32], a two-qubit state is assumed to be

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|u\rangle|z+\rangle + |d\rangle|z-\rangle), \tag{1}$$

where $|u\rangle$ and $|d\rangle$ mean paths, 'up' and 'down', $|z+\rangle$ and $|z-\rangle$ mean the polarization states, 'vertical' and 'horizontal'. The four observables Z_1, Z_2, X_1, and X_2, where the subscripts 1 and 2 denote the path and

*Supported by the National Natural Science Foundation of China under Grant Nos 10825416 and 60708009, the National High Technology Research and Development Program of China (No 2011AA120102).
**Corresponding author. Email: zhangxd@bit.edu.cn; zhangxd@bnu.edu.cn
© 2012 Chinese Physical Society and IOP Publishing Ltd

polarization qubits, are expressed by

$$\begin{aligned} Z_1 &= |u\rangle\langle u| - |d\rangle\langle d|, \\ X_1 &= |u'\rangle\langle u'| - |d'\rangle\langle d'|, \\ Z_2 &= |z+\rangle\langle z+| - |z-\rangle\langle z-|, \\ X_2 &= |x+\rangle\langle x+| - |x-\rangle\langle x-|, \end{aligned} \tag{2}$$

where $|u'\rangle = (1/\sqrt{2})(|u\rangle + |d\rangle)$, $|d'\rangle = (1/\sqrt{2})(|u\rangle - |d\rangle)$, $|x+\rangle = (1/\sqrt{2})(|z+\rangle + |z-\rangle)$ and $|x-\rangle = (1/\sqrt{2})(|z+\rangle - |z-\rangle)$. According to the above experimental setup and the definition of states, if the results of Z_1X_2 and X_1Z_2 are same, they are in agreement with the NCHV theories. If the results of Z_1X_2 and X_1Z_2 are opposite, they are consistent with quantum mechanics. In Ref. [32], the required quantum state is obtained by a properly rotated half-wave plate and a polarizing beam splitter. The results of Z_1Z_2 and X_1X_2 are both $+1$, and the results of the joint measurement of Z_1X_2 and X_1Z_2 are opposite, so their experimental results are in agreement with quantum mechanics.

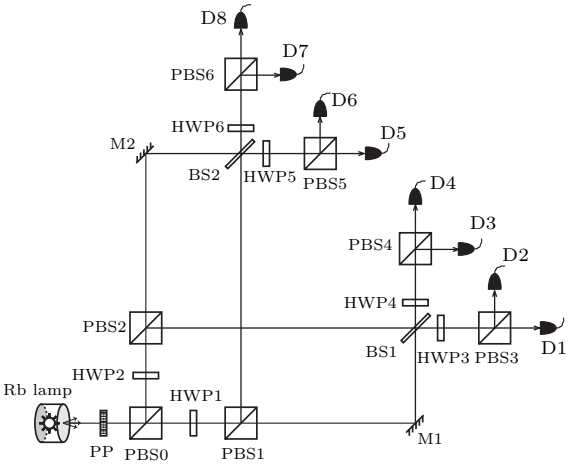

Fig. 1. Experimental setup to test the Kochen–Specker theorem with thermal light. The required light is acquired by a properly rotated polarizing plate (PP) and a polarizing beam splitter (PBS0). The two half-wave plates (HWP1 and HWP2) are both set at 0° (45°) for setup 1. HWP1 is set at 22.5° and HWP2 is set at −67.5° in setup 2. The other half-wave plates are all set at 22.5°. M1 and M2 are mirrors, D1–D8 are detectors.

Now we use the classical thermal light source to perform the similar experiment. In our experiment, the definitions of Z_1, Z_2, X_1, and X_2 are similar to those in Ref. [32]. They are assumed to be: (a) When the light passes through PBS0, in which path we find it, up ($Z_1 = +1$) or down ($Z_1 = -1$). (b) The polarization of the light, vertical ($Z_2 = +1$) or horizontal ($Z_2 = -1$). (c) When the up and the down path lengths between PBS0 and BS1 are equal, in which path we find the light after BS1 (BS2), up ($X_1 = +1$) or down ($X_1 = -1$). (d) The polarization of the light away from the horizontal direction, +45° ($X_2 = +1$) or −45° ($X_2 = -1$). According to the above definition, it is obvious that $Z_1Z_2 = +1$. In the following, we perform the experiment to study the properties of X_1X_2, Z_1X_2 and X_1Z_2.

For the symmetry of the setup in Fig. 1, we first discuss the interferometer formed by PBS0 and the beam splitter (BS1) in detail. We tune the interferometer by PBS0 and BS1 to arm-equaled to perform the measurement of X_1 with HWP1 and HWP2 both at 0°. We set both HWP3 and HWP4 at 22.5° followed by PBS3 and PBS4 to measure X_2. When they work together, the measurement of X_1X_2 can be accomplished. This setup is named setup 1. The recorded results are shown in the first stage (0–60 s) in Fig. 2. At such a stage, we can obtain the minimal counts at D1, which means maximal counts at D2. The maximal counts at D2 means that the light is in the 'down' path after BS1, that is, $X_1 = -1$. Also we can conclude that the polarization of the light is −45°, and this means $X_2 = -1$. Thus we obtain the result $X_1X_2 = +1$. The same analysis is available to D3 and D4. For the interferometer formed by PBS0 and BS2, we just set both HWP1 and HWP2 at +45°, at the same time, keep this interferometer arm-equaled, HWP5 and HWP6 are both set at 22.5°, the corresponding results can also be obtained.

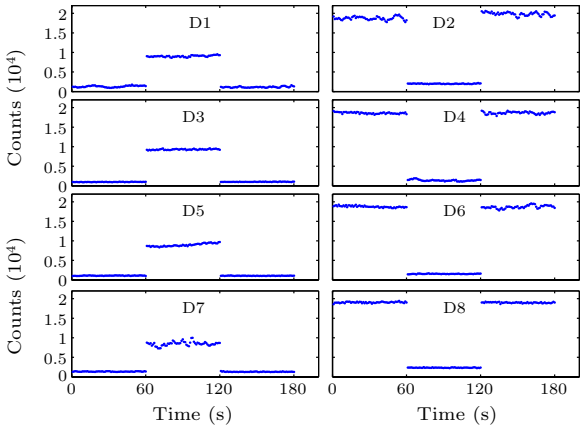

Fig. 2. (Color online) Counts of D1–D8 in the experiment. The vertical axis is the counts of single photon detectors and the horizontal axis is the time axis (in units of second) in the experiment. The first stage (0–60 s) and the third stage (120–180 s) both demonstrate the results of setup 1. The second stage (60–120 s) demonstrates the results of setup 2.

Every curve in Fig. 2 includes three stages along the time axis. The second stage (60–120 s) demonstrates the results of setup 2. For such a case, we set HWP1 and HWP2 at 22.5° and −67.5°, respectively, and set both HWP3 and HWP4 at 22.5° (with PBS3 and PBS4) to perform the joint measurement Z_1X_2 and X_1Z_2. According to such a process, we find that the maximal counts is reached at D1, D3, D5, and D7 (minimal counts at D2, D4, D6, and D8) as shown in Fig. 2. The maximal counts at D1 means that the polarization up path is −45° after PBS0, $Z_1 = +1$ and $X_2 = -1$; the polarization down path is 45°, $Z_1 = -1$ and $X_2 = +1$. That is, $Z_1X_2 = -1$. In contrast, $X_1Z_2 = +1$ after BS1. Thus, Z_1X_2 and X_1Z_2 are op-

posite. This means that our experimental results are incompatible with the NCHV theories, but in agreement with quantum mechanics.

If we tune the setup to setup 1 again, the recorded results are plotted in the third stage (120–180 s) in Fig. 2, which is in agreement with those in the first stage. This means that our system is stable. In our experiment, we did not record the eight detectors D1–D8 at the same time for the limitation of our devices. However, the eight cases are nearly the same. Our experimental results are similar to those in Ref. [32] although we use the classical thermal light source instead of quantum light sources, which can be understood in the following.

When the electric field of the properly prepared input beam $\boldsymbol{E}$ passes through the polarizing beam splitter PBS0, it changes to

$$\boldsymbol{E} \to \frac{1}{\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,d}} + \boldsymbol{E}_{\mathrm{V,u}}), \tag{3}$$

where $|\boldsymbol{E}_{\mathrm{H,d}}| = |\boldsymbol{E}_{\mathrm{V,u}}| = |\boldsymbol{E}|$, the subscripts H and V denote the polarization, 'horizontal' and 'vertical', and the subscripts d and u denote pathes, 'down' and 'up', respectively. We know that the role of the 50/50 beam splitter can be described by Jones matrix $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.[37] Thus, when HWP1 and HWP2 are set at 0°, after the incident field passes through the interferometer performed by PBS0 and BS1 (setup1), it is expressed as

$$\begin{aligned} \boldsymbol{E} &\to \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{H,d}} + \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{V,u}} \to \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{H,d}} - \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{V,u}} \\ &\to \frac{1}{2}(\boldsymbol{E}_{\mathrm{H,d'}} - \boldsymbol{E}_{\mathrm{H,d'}}) + \frac{1}{2}(\boldsymbol{E}_{\mathrm{H,u'}} + \boldsymbol{E}_{\mathrm{V,u'}}), \end{aligned} \tag{4}$$

where $|\boldsymbol{E}_{\mathrm{H,d'}}| = |\boldsymbol{E}_{\mathrm{H,d'}}| = |\boldsymbol{E}_{\mathrm{H,u'}}| = |\boldsymbol{E}_{\mathrm{V,u'}}|\,(= |\boldsymbol{E}|)$ represent the fields after BS1, the subscripts d' and u' also denote path 'down' and 'up'. Then, the beams from the BS1 pass through HWP3 (22.5°) and HWP4 (22.5°), they turn to

$$\begin{aligned} &\frac{1}{2}(\boldsymbol{E}_{\mathrm{H,d'}} - \boldsymbol{E}_{\mathrm{H,d'}}) \to \frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,d'}} + \boldsymbol{E}_{\mathrm{H,d'}}) \\ &- \frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,d'}} - \boldsymbol{E}_{\mathrm{H,d'}}) = \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{H,d'}}, \\ &\frac{1}{2}(\boldsymbol{E}_{\mathrm{H,u'}} + \boldsymbol{E}_{\mathrm{V,u'}}) \to \frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,u'}} + \boldsymbol{E}_{\mathrm{V,u'}}) \\ &+ \frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,u'}} - \boldsymbol{E}_{\mathrm{V,u'}}) = \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{H,u'}}. \end{aligned} \tag{5}$$

From Eq. (5), we can see that the polarization of light is vertical in the down path and horizontal in the up path after BS1. Thus, we get the maximal counts at D2 and D4 (minimal at D1 and D3). Such an analysis is also suitable for the interferometer performed by PBS0 and BS2 and other detectors (D5 and D7, D6 and D8).

When the joint measurements of Z_1X_2 and X_1Z_2 are performed by rotating HWP1 and HWP2 to 22.5° and −67.5°, respectively, the electric field $\boldsymbol{E}$ is expressed as

$$\begin{aligned} \boldsymbol{E} &\to \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{H,d}} + \frac{1}{\sqrt{2}}\boldsymbol{E}_{\mathrm{V,u}} \to \frac{1}{2}(\boldsymbol{E}_{\mathrm{H,d}} + \boldsymbol{E}_{Vd}) \\ &+ \frac{1}{2}(-\boldsymbol{E}_{\mathrm{H,u}} + \boldsymbol{E}_{\mathrm{V,u}}) \to \frac{1}{2}\boldsymbol{E}_{\mathrm{H,d}} + \frac{1}{2}\boldsymbol{E}_{\mathrm{V,u}} \\ &\to \frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,d'}} + \boldsymbol{E}_{\mathrm{H,d'}}) + \frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,u'}} - \boldsymbol{E}_{\mathrm{V,u'}}). \end{aligned} \tag{6}$$

The electric fields output from BS1 turn to

$$\begin{aligned} &\frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,d'}} + \boldsymbol{E}_{\mathrm{H,d'}}) \to \frac{1}{4}(\boldsymbol{E}_{\mathrm{H,d'}} + \boldsymbol{E}_{\mathrm{H,d'}}) \\ &+ \frac{1}{4}(\boldsymbol{E}_{\mathrm{H,d'}} - \boldsymbol{E}_{\mathrm{H,d'}}) = \frac{1}{2}\boldsymbol{E}_{\mathrm{H,d'}}, \\ &\frac{1}{2\sqrt{2}}(\boldsymbol{E}_{\mathrm{H,u'}} - \boldsymbol{E}_{\mathrm{V,u'}}) \to \frac{1}{4}(\boldsymbol{E}_{\mathrm{H,u'}} + \boldsymbol{E}_{\mathrm{V,u'}}) \\ &- \frac{1}{4}(\boldsymbol{E}_{\mathrm{H,u'}} - \boldsymbol{E}_{\mathrm{V,u'}}) = \frac{1}{2}\boldsymbol{E}_{\mathrm{V,u'}}. \end{aligned} \tag{7}$$

Thus, we obtain maximal counts at D1 and D3 (minimal at D2 and D4). Similarly, we get maximal counts at D5 and D7 (minimal at D6 and D8) through the interferometer performed by PBS0 and BS2. These counts in setup 2 are half of those in setup 1. From the above analyses, it is not difficult to find that the experimental phenomena originate from the coherent superposition properties of light in the interferometers, which do not depend on whether the states used in the experiment are quantum or classical.

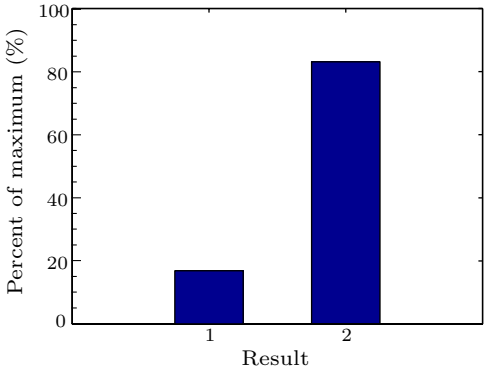

Fig. 3. (Color online) Analyzed experimental results. Results 1 and 2 are the fractions which are in agreement with the NCHV theories and quantum mechanics, respectively. The results demonstrate the violation against NCHV theories in our experiment.

If we introduce a fraction number ε as in Ref. [32], we can also demonstrate the above deduction from the analyzed experimental results. In our experiment, $\varepsilon = S_{\min}/(S_{\max} + S_{\min}) = 0.17$, where $S_{\min}$ is the sum of the average minimum coincidence counts of D2, D4, D6 and D8, and $S_{\max}$ is the sum of the average maximum coincidence counts of D1, D3, D5 and D7. The analyzed experimental results are plotted in Fig. 3. Results 1 and 2 are the fractions which agree with the NCHV theories and quantum mechanics, re-

spectively. Our experimental results demonstrate the violation of NCHV theories.

In summary, we have performed an all-or-nothing–type KS experiment using the thermal light source instead of other quantum light sources, and obtained the same results as those using quantum light sources. Although the thermal-light exhibits similar phenomena to the single photon in the first-order interference it contains multiple photon numbers which cannot be treated as a single-photon Fock state. It obeys a Gaussian distribution and can not be regarded as the simple summation of single photons. Therefore, our experimental results bring up an interesting problem: how to clarify the differences between quantum mechanics and NCHV theories without using the single-photon Fock state. It is evident that such a study can deepen our understanding of the basic physical problem. However, it is complex to construct NCHV theories by using multiple photon states, which is worthy of study in the future.

References

[1] Bennink R S, Bentley S J and Boyd R W 2002 *Phys. Rev. Lett.* **89** 113601
[2] Gatti A Brambilla E, Bache M and Lugiato L A 2004 *Phys. Rev. Lett.* **93** 093602
[3] Cheng J and Han S S 2004 *Phys. Rev. Lett.* **92** 093903
[4] Cai Y J and Zhu S Y 2005 *Phys. Rev.* E **71** 056607
[5] Cao D Z Xiong J and Wang K G 2005 *Phys. Rev.* A **71** 013801
[6] Valencia A, Scarcelli G, D'Angelo M and Shih Y 2005 *Phys. Rev. Lett.* **94** 063601
[7] Ferri F, Magatti D, Gatti A, Bache M, Brambilla E and Lugiato L A 2005 *Phys. Rev. Lett.* **94** 183602
[8] Klyshko D N 1988 *Sov. Phys. JETP* **67** 1131
Klyshko D N 1988 *Photon and Nonlinear Optics* (New York: Gordon and Breach Science)
[9] Pittman T B, Shih Y H, Strekalov D V and Sergienko A V 1995 *Phys. Rev.* A **52** R3429
[10] Cao D Z, Xiong J and Wang K G 2005 *Phys. Rev.* A **71** 013801
[11] Scarcelli G, Berardi V and Shih Y H 2006 *Appl. Phys. Lett.* **88** 061106
2006 *Phys. Rev. Lett.* **96** 063602
[12] Chen X H, Liu Q, Luo K H and Wu L A 2009 *Opt. Lett.* **34** 695
[13] Shen X, Bai Y F, Qin T and Han S S 2008 *Chin. Phys. Lett.* **25** 3968
[14] Aspect A, Dalibard J and Roger G 1982 *Phys. Rev. Lett.* **49** 1804
[15] Rarity J G and Tapster P R 1990 *Phys. Rev. Lett.* **64** 2495
[16] Weihs G, Jennewein T, Simon C Weinfurter H and Zeilinger A 1998 *Phys. Rev. Lett.* **81** 5039
[17] Tittel W, Brendel J, Zbinden H and Gisin N 1998 *Phys. Rev. Lett.* **81** 3563
[18] Rowe M A, Kielpinski D Meyer, V Sackett C A, Itano W M Monroe C and Wineland D J 2001 *Nature* **409** 791
[19] Hasegawa Y, Loidl R, Badurek G, Baron M and Rauch H 2003 *Nature* **425** 45
[20] Groblacher S, Paterek T, Kaltenbaek R, Brukner C, Zukowski M, Aspelmeyer M and Zeilinger A 2007 *Nature* **446** 871
[21] Matsukevich D N, Maunz P, Moehring D L, Olmschenk S and Monroe C 2008 *Phys. Rev. Lett.* **100** 150404
[22] Kochen S and Specker E P 1967 *J. Math. Mech.* **17** 59
[23] Tang J S, Li C F, Xu J S, Chen G, Zou Y, Zhou Z Q and Guo G C 2011 *Chin. Phys. Lett.* **28** 060304
[24] Gao S J and Fu J 2008 *Chin. Phys. Lett.* **25** 2350
[25] Peng L, Huang Y F, Li L, Liu B H, Li C F and Guo G C 2011 *Chin. Phys. Lett.* **28** 070308
[26] Mermin N D 1993 *Rev. Mod. Phys.* **65** 803
[27] Cabello A and Garcia-Alcaine G 1996 *J. Phys.* A **29** 1025
Cabello A, Estebaranz J M and Garcia-Alcaine G 1996 *Phys. Lett.* A **212** 183
Cabello A, Estebaranz J M and Garcia-Alcaine G 1996 *Phys. Lett.* A **218** 115
[28] Cabello A and Garcia-Alcaine G 1998 *Phys. Rev. Lett.* **80** 1797
[29] Cabello A 2001 *Phys. Rev. Lett.* **87** 010403
[30] Simon C, Zukowski M, Weinfurter H and Zeilinger A 2000 *Phys. Rev. Lett.* **85** 1783
[31] Cabello A 2008 *Phys. Rev. Lett.* **101** 210401
[32] Huang Y F Li C F, Zhang Y S, Pan J W and Guo G C 2003 *Phys. Rev. Lett.* **90** 250401
[33] Amselem E, Radmark M, Bourennane M and Cabello A 2009 *Phys. Rev. Lett.* **103** 160405
[34] Hasegawa Y, Loidl R, Badurek G Baron M and Rauch H 2006 *Phys. Rev. Lett.* **97** 230401
[35] Bartosik H, Klepp J, Schmitzer C, Sponar S, Cabello A Rauch H and Hasegawa Y 2009 *Phys. Rev. Lett.* **103** 040403
[36] Michler M, Weinfurter H and Zukowski M 2000 *Phys. Rev. Lett.* **84** 5457
[37] Kirchmair G, Zaehringer F, Gerritsma R, Kleinmann M, Guehne O, Cabello A, Blatt R and Roos C F 2009 *Nature* **460** 494
[38] Bachor H A and Ralph T C 2004 *A Guide to Experiments in Quantum Optics* 2nd edn (New York: WILEY-VCH Verlag) pp 99–104