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Second-Order Contribution of the Incompressibility in Asymmetric Nuclear Matter *

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With the complementarity of the nucleonic three-body force, we present the saturation points of symmetric nuclear matter with different interactions adopted within the Brueckner–Hartree–Fock scheme, and a more accurate empirical parameterization function for the equation of state of symmetric nuclear matter and pure neutron matter. On the basis of this fit formula, the symmetry energy and its derivatives are investigated, and ultimately the higher-order coefficient of the isobaric incompressibility for isospin asymmetric nuclear matter is predicted.

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With the advent and development of many radioactive beam facilities around the world, research of the nuclear equation of state (EOS) of asymmetric nuclear matter under the extreme condition of large isospin asymmetry becomes a more crucial issue studied not only in nuclear physics but also in nuclear astrophysics. Various microscopic and phenomenological approaches, such as Brueckner–Hartree–Fock (BHF),[1–3] Dirac-BHF,[4,5] the variational method,[6,7] the relativistic mean field,[8,9] and the Skymre–Hartree–Fock model,[10] have been employed to explore the isospin-dependent properties of asymmetric nuclear matter, especially with regard to the density dependence of the nuclear symmetry energy and the incompressibility. The considerable effort[11,12] made to produce dynamical simulations of collisions between neutron-rich nuclei shows that the structure of radioactive nuclei as well as the reaction dynamics is quite sensitive to the density dependence of the nuclear symmetry energy. However, nowadays, the theoretical knowledge of the incompressibility of asymmetric nuclear matter at its saturation density is much poorer, which is quite undetermined and any constraint imposed on it would be extraordinarily important.

It is well known that in the framework of BHF scheme, nucleonic three-body force (TBF) is quite indispensable to reproduce the empirical saturation properties of symmetric nuclear matter. Particularly, in the high densities, TBF plays an important or even dominant role in the complementarity of the effective in-medium nucleon-nucleon interaction that is used as an input to the above theoretical method. Firstly, the microscopic model of TBF was introduced in Ref. [13], which was entirely based on the meson-exchange picture of the nucleon-nucleon interaction and involves the intermediate excitation of Delta(1232), Roper(1440), and nucleon-antinucleon states by the exchange of $\pi, \rho, \sigma, \omega$ mesons in the TBF diagrams. Recently, a new construction is presented in Ref. [14] to treat two- and three-body forces on the same theoretical footing, i.e., the same microscopical meson-exchange parameters are chosen in both two- and three-body forces. It was shown that the saturation point of nuclear matter is significantly improved compared with the use of a two-body force alone.

In this Letter, based on the study in Refs. [14,15], we detail the information on the construction of the microscopic TBF, and display the higher-order expansion of symmetry energy and the incompressibility at the saturation density for asymmetric nuclear matter. Now we begin with a brief review of the theoretical framework.

For the sake of avoiding the difficulty of solving the relevant Faddeev equations in the use of BHF calculation, the TBF is reduced to an effective, density-dependent, two-body force by averaging over the third nucleon in the medium,

$$V_3 = \overline{V}_{12}(\mathbf{r}) = \rho \int d^3 r_3 \sum_{\sigma_3,\tau_3} g(r_{13})^2 g(r_{23})^2 V_{123}, \quad (1)$$

where g is called the defect function, which

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takes account of the nucleon-nucleon in-medium correlations[13,16,17] and is added to the two-body bare potential in the following Bethe–Goldstone equation,[18]

$$G[\omega;\rho] = V + \sum_{k_a,k_b\rangle k_F} V \frac{|\ k_a k_b\rangle\langle k_a k_b\ |}{\omega - e(k_a) - e(k_b) + i\epsilon} G[\omega;\rho]. \tag{2}$$

In this equation the Brueckner reaction matrix G is the basic ingredient, and $V = V_2 + V_3$ is the nucleon-nucleon interaction composed of two-nucleon potential V_2 and averaged TBF V_3 from Eq. (1), ρ is the nucleon number density, and ω means the starting energy. The propagation of intermediate nucleon pairs is constrained above the Fermi momentum k_F, and the nucleon single-particle energy is

$$e(k) = e(k;\rho) = \frac{k^2}{2m} + U(k;\rho). \tag{3}$$

The BHF approximation for the single-particle potential $U(k;\rho)$ using the continuous choice prescription is expressed as

$$U(k;\rho) = Re \sum_{k'\le k_F} \langle kk'|G[e(k) + e(k');\rho]|kk'\rangle_A, \tag{4}$$

where the subscript A indicates anti-symmetrization of the matrix elements. It has been demonstrated that the nuclear EOS can be calculated with good accuracy in the BHF two-hole-line approximation with the continuous choice for the single-particle potential.[19,20]

Equations (2)–(4) constitute a coupled system that needs to be solved self-consistently together with the computation of the defect function and the averaged TBF given in Eq. (1). In the BHF approximation, technically, we have tried to obtain accurate results by enforcing the self-consistency condition (4) up to $k = 9\,\mathrm{fm}^{-1}$ in momentum space, and by including a large number of partial waves ($J_{\max} = 9$) in the expansion of the G-matrix. Thus the energy per nucleon is derived by

$$\begin{aligned} E = \frac{B}{A} =& \frac{3}{5}\frac{k_F^2}{2m} + \frac{1}{2\rho} Re \sum_{k,k'\le k_F} \langle kk'|G[e(k) \\ &+ e(k');\rho]|kk'\rangle_A. \end{aligned} \tag{5}$$

Following the spirit of the literature,[21] at the nuclear matter saturation density ρ_0 the binding energy per particle for symmetric nuclear matter $E(\rho)$ can be expanded as

$$E(\rho) = E_0(\rho_0) + J_0\chi + \frac{K_0}{2!}\chi^2 + \frac{L_0}{3!}\chi^3 + \cdots, \tag{6}$$

where $\chi = \frac{\rho-\rho_0}{3\rho_0}$ denotes the deviation of the density from the saturation density of symmetric nuclear matter, and the coefficients J_0, K_0, L_0 are defined by

$$J_0 = 3\rho_0 \frac{dE(\rho)}{d\rho}|_{\rho=\rho_0} = 0, \tag{7}$$

$$K_0 = 9\rho_0^2 \frac{d^2E(\rho)}{d\rho^2}|_{\rho=\rho_0}, \tag{8}$$

$$L_0 = 27\rho_0^3 \frac{d^3E(\rho)}{d\rho^3}|_{\rho=\rho_0}. \tag{9}$$

In the same way, the nuclear symmetry energy $E_{\rm sym}(\rho)$ can be spread as

$$\begin{aligned} E_{\rm sym}(\rho) =& E_{\rm sym}(\rho_0) + E_{{\rm sym},1}\chi \\ &+ \frac{E_{{\rm sym},2}}{2!}\chi^2 + \frac{E_{{\rm sym},3}}{3!}\chi^3 + \ldots, \end{aligned} \tag{10}$$

where $E_{{\rm sym},1}$, $E_{{\rm sym},2}$, $E_{{\rm sym},3}$ are characterized by

$$E_{{\rm sym},1} = 3\rho_0 \frac{dE_{\rm sym}(\rho)}{d\rho}\Big|_{\rho=\rho_0}, \tag{11}$$

$$E_{{\rm sym},2} = 9\rho_0^2 \frac{d^2E_{\rm sym}(\rho)}{d\rho^2}\Big|_{\rho=\rho_0}, \tag{12}$$

$$E_{{\rm sym},3} = 27\rho_0^3 \frac{d^3E_{\rm sym}(\rho)}{d\rho^3}\Big|_{\rho=\rho_0}. \tag{13}$$

For the isospin asymmetric nuclear matter, the incompressibility is as a function of the isospin asymmetry. At the saturation density, it is usually called the isobaric incompressibility coefficient[21,22] and is written as

$$K(\beta) = K_0 + K_2\beta^2 + K_4\beta^4 + \ldots, \tag{14}$$

where $\beta = \frac{\rho_n-\rho_p}{\rho}$ with ρ_n and ρ_p denoting the neutron and proton densities, and K_2 reads[21]

$$K_2 = E_{{\rm sym},2} - 6E_{{\rm sym},1} - \frac{L_0}{K_0}E_{{\rm sym},1}. \tag{15}$$

For convenience of the further discussion, we firstly demonstrate the effect brought by TBF on the EOS of nuclear matter. In Fig. 1 the saturation points of symmetric nuclear matter extracted from the different modern interactions such as Paris,[13] Argonne V_{18} (Av18),[23] Bonn B[24] and Nijmegen93 (Nij93)[25] potentials with and without the corresponding TBF complemented are presented. Here it should be pointed out that the meson-exchange parameters adopted by the TBFs are chosen completely consistent with the ones used in their respective two-body potentials. From this figure, it can be seen clearly that the saturation properties have been improved evidently no matter which interaction is adopted within BHF approach. Among them the result from the Bonn B potential plus TBF is completely shifted to the empirical area (shaded region), and the calculations from other three potentials are also close to the empirical region. Of course, one should not expect all perfect reproductions of the empirical saturation point from different nucleon-nucleon interactions

on account of the numerous theoretical and numerical approximations involved in both the construction of the effective TBF and the approximation of BHF approach.

Compared with those in Ref. [15] a more accurate empirical parametrization of the functional form for the EOS of symmetric nuclear matter and pure neutron matter is shown,

$$E(\rho) = a\rho + b\rho^c + d, \tag{16}$$

where the highest density required to fit the formula is restricted to $\rho = 0.6\,\mathrm{fm}^{-3}$, not the same as the value in the literature[15] just for the investigation on the dense stellar objects. The fit parameters a, b, c, d for symmetric and neutron matter are listed in Table 1, which allow us to reproduce the initial data within 1 MeV approximation.

Table 1. Parameters of the EOS for symmetric nuclear matter and pure neutron matter using different interactions.

	Symmetric matter				Neutron matter			
	a	b	c	d	a	b	c	d
Paris	−99.1	430.5	2.53	−3.19	33.7	644.7	2.87	4.78
Av18	−101.3	371.0	2.39	−3.12	32.5	585.7	2.73	4.87
Bonn B	−151.4	511.3	2.13	−2.21	21.8	814.0	2.83	3.79
Nij93	−242.6	385.7	1.52	−0.21	39.5	712.3	2.55	4.0

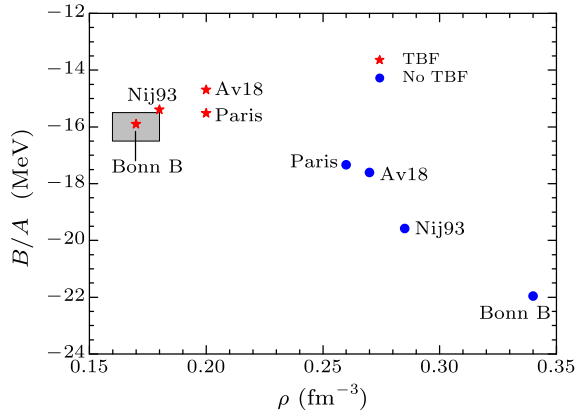


Fig. 1. Saturation points of symmetric nuclear matter obtained with different potentials. The asterisks represent the results with TBF, and the dots without TBF effect. The shadow square indicates the empirical region.

Taking advantage of these simple fits, in Table 2 we display the symmetry energy (second column), its first (third column) and second derivatives (fourth column), and the incompressibility (fifth column) at the normal density of symmetric nuclear matter besides the coefficient K_2 (sixth column) given in Eq. (15) which reflects the isospin dependence of the isobaric incompressibility. We can see that nearly all the interactions produce reasonable values for these physical quantities aside from the Nij93 with slightly lower incompressibility compared with the empirical estimate $K_0 = 240 \pm 20$ MeV. The symmetry energy and its first-order derivative in all the cases are consistent with the constraints $E_{\mathrm{sym}}(\rho_0) = 30 \pm 5$ MeV and 46 MeV $\leq E_{\mathrm{sym},1} \leq 111$ MeV obtained from the analysis of isospin diffusion in the heavy-ion reactions and the neutron skin of ^{208}Pb proposed in Ref. [26], and also the second-order derivative of symmetry energy is compatible with the prediction from the Ref. [21]. However, some of the obtained data are slightly different to the ones in Ref. [15] as a result of the different limitation of the highest density fitted and the selection of the fit function.

Table 2. The first and second order character parameters in the expansion of symmetry energy and impressibility at the saturation density.

	E_{sym}	$E_{\mathrm{sym},1}$	$E_{\mathrm{sym},2}$	K_0	K_2
Paris	31.0	69.5	31.1	257	−495
Av18	31.3	82.7	70.7	236	−523
Bonn B	30.3	59.2	−2.3	254	−381
Nij93	35.1	91.3	117.2	202	−299

For more distinction, in Fig. 2 we demonstrate the coefficient K_2 in the expansion of the isobaric incompressibility of Eq. (14) as a function of $E_{\mathrm{sym},1}$. In this figure it can be seen that the Paris and Av18 potentials yield the same value as the constraint of $K_2 = -550 \pm 100$ MeV extracted from the recent measurements of the isotopic dependence of the GMR in even-A Sn isotopes,[27,28] whereas the results from Bonn B and Nij93 interactions are consistent with the estimated value of $K_2 = -370 \pm 120$ MeV from a phenomenological modified Skyrme-like model.[21]

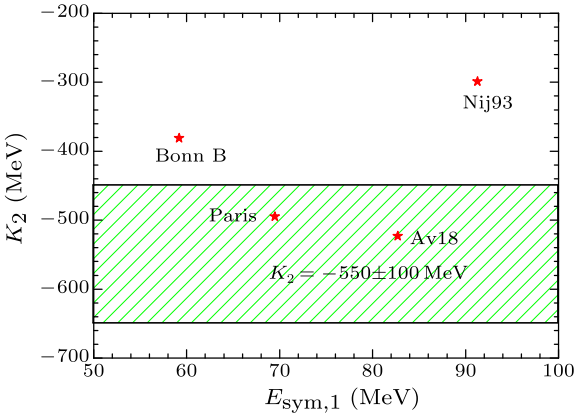


Fig. 2. The coefficient K_2 of the isobaric incompressibility as a function of the slope parameter $E_{\mathrm{sym},1}$ of symmetry energy derived from different interactions complemented by TBF, compared with the measurements of the GMR of Sn isotopes[27,28] indicated by the shadow area.

In summary, we have shown the saturation properties of symmetric nuclear matter with various interactions plus the TBF effect in the framework of the BHF approach. A more accurate empirical parametrization function is presented for the equation of state of the symmetric nuclear matter and pure neutron matter. On the basis of this formula, special attention is paid

to the higher orders of symmetry energy and the isobaric incompressibility. In our cases, the symmetry energy and its first-order derivative both are compatible with the constraints $E_{\rm sym}(\rho_0) = 30 \pm 5\,{\rm MeV}$ and $46\,{\rm MeV} \leq E_{\rm sym,1} \leq 111\,{\rm MeV}$ obtained from the heavy-ion collision data. However, the Nij93 gives a prediction with a slightly lower incompressibility compared with the empirical estimate $K_0 = 240 \pm 20\,{\rm MeV}$. With respect to the coefficient K_2 denoting the isospin dependence of the isobaric incompressibility for isospin asymmetric nuclear matter, using the Paris and Av18 potentials plus the respective TBF we derive the consistent results with the constraint of $K_2 = -550 \pm 100\,{\rm MeV}$ extracted from GMR of Sn Isotopes.

References

[1] Baldo M, Bombaci I and Burgio G F 1997 *Astron. Astrophys.* **328** 274
[2] Lejeune A, Lombardo U and Zuo W 2000 *Phys. Lett.* B **477** 45
[3] Zuo W, Lejeune A, Lombardo U and Mathiot J F 2002 *Nucl. Phys.* A **706** 418
[4] Brockmann R and Machleidt R 1990 *Phys. Rev.* C **42** 1965
[5] Alonso D and Sammarruca F 2003 *Phys. Rev.* C **67** 054301
[6] Carlson J, Pandharipande V R and Wiringa R B 1983 *Nucl. Phys.* A **401** 59
[7] Akmal A, Pandharipande V R and Ravenhall D G 1998 *Phys. Rev.* C **58** 1804
[8] Ren Z Z, Tai F and Chen D H 2002 *Phys. Rev.* C **66** 064306
[9] Meng J, Toki H, Zhou S G et al 2006 *Prog. Part. Nucl. Phys.* **57** 470
[10] Chen L W, Ko C M and Li B A 2005 *Phys. Rev.* C **72** 064309
[11] Di Toro M, Baran V, Colonna M et al 1999 *Prog. Part. Nucl. Phys.* **42** 125
[12] Li B A, Chen L W and Ko C M 2008 *Phys. Rep.* **464** 113
[13] Grangé P, Lejeune A, Martzolff M and Mathiot J F 1989 *Phys. Rev.* C **40** 1040
[14] Li Z H, Lombardo U, Schulze H J and Zuo W 2008 *Phys. Rev.* C **77** 034316
[15] Li Z H and Schulze H J 2008 *Phys. Rev.* C **78** 028801
[16] Coon S A, Scadron M D, McNamee P C et al 1979 *Nucl. Phys.* A **317** 242
[17] Baldo M and Ferreira L S 1999 *Phys. Rev.* C **59** 682
[18] Jeukenne J P, Lejeune A and Mahaux C 1976 *Phys. Rep.* **25** 83
[19] Song H Q, Baldo M, Giansiracusa G and Lombardo U 1998 *Phys. Rev. Lett.* **81** 1584
[20] Sartor R 2006 *Phys. Rev.* C **73** 034307
[21] Chen L W, Cai B J, Ko C M et al 2009 *Phys. Rev.* C **80** 014322
[22] Prakash M and Bedell K S 1985 *Phys. Rev.* C **32** 1118
[23] Wiringa B R, Stoks V G J and Schiavilla R 1995 *Phys. Rev.* C **51** 38
[24] Machleidt R, Holinde K and Elster C 1987 *Phys. Rep.* **149** 1
Machleidt R 1989 *Adv. Nucl. Phys.* **19** 189
[25] Stoks V G J, Klomp R A M, Terheggen C P F and de Swart J J 1994 *Phys. Rev.* C **49** 2950
[26] Li B A and Steiner A W 2006 *Phys. Lett.* B **642** 436
[27] Li T et al 2007 *Phys. Rev. Lett.* **99** 162503
[28] Garg U et al 2007 *Nucl. Phys.* A **788** 36