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Size Segregation in Rapid Flows of Inelastic Particles with Continuous Size Distributions *

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Two-dimensional numerical simulations are employed to gain insight into the segregation behavior of granular mixtures with a power-law particle size distribution in the presence of a granular temperature gradient. It is found that particles of all sizes move toward regions of low granular temperature. Species segregation is also observed. Large particles demonstrate a higher affinity for the low-temperature regions and accumulate in these cool regions to a greater extent than their smaller counterparts. Furthermore, the local particle size distribution maintains the same form as the overall (including all particles) size distribution.

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Rapid granular flows characterized by nearly instantaneous collisions between grains occur in both nature and industry. Granular flows exhibit a host of complex behaviors such as segregation among unlike particles.[1−4] Industrially, segregation among unlike particles can be either beneficial (e.g., separation processes) or detrimental (e.g., mixing processes) depending on the desired product properties. Segregation is observed in both slow granular flows and rapid granular flows. For the case of rapid granular flows, segregation generally results from spatial variations in the granular temperature (i.e., the kinetic energy associated with the random motions of the particles) and the application of external forces.[5,6] To date, segregation studies within the rapid granular flow regime have been targeted at binary systems. Hsiau and Hunt[6] used the kinetic theory of Jenkins and Mancini[7] to study the segregation in a system with zero mean flow in the presence of a granular temperature gradient. Luding *et al.*[8] performed molecular-dynamics simulations to reveal the segregation behavior of inelastic, binary mixtures exposed to two independent granular temperature sources. Louge *et al.*[9] performed experiments in reduced gravity to study segregation in rapid granular flows with binary mixtures. The aforementioned investigations suggest that all particles prefer regions of lower granular temperature but large particles appear to have a higher affinity for cold regions. In addition, complete segregation is possible at high granular temperature gradients or large mass differences between unlike particles. However, natural and industrial granular flows often contain a continuous particle size distribution, whereas the majority of previous efforts for mixtures have been focused on binary systems. The segregation behavior of granular mixtures with continuous size distributions in rapid flows has seldom been reported.

In this Letter, molecular-dynamics simulations are employed to study size segregation of granular mixtures with a continuous size distribution in the presence of a granular temperature gradient. Spatial profiles of the solid area fraction ϕ, granular temperature T and local size distributions are reported. The results indicate that particles of all sizes move preferentially toward areas of low granular temperature. Large particles are especially concentrated in areas of low granular temperature. The local particle size distribution remains of the same type as the initial overall (including all particles in the domain) particle size distribution. The mean diameter of the local size distribution increases as the granular temperature decreases. Complete segregation is not observed even at extremely large granular temperature ratios.

In our event-driven simulations, particles are confined to motion in a two-dimensional square box, as shown in Fig. 1. The length of the simulation domain in the x and y directions are H_x and H_y, respectively. The simulation domain is bounded on the top and bottom by solid walls of "set" granular temperature $T_{\rm set}$. The particles are given a post-collision velocity from a Maxwellian distribution that is consistent with $T_{\rm set}$ by colliding with the wall. The left and right boundaries of the simulation domain are periodic.

Collisions between particles are treated as hard-sphere collisions (instantaneous and binary). Linear momentum is conserved during the collisions while kinetic energy is dissipated via inelastic collisions. When two particles i and j collide, their post-collisional velocities depend on their pre-collisional velocities in the following way:

$$\boldsymbol{v}_{i,post} = \boldsymbol{v}_{i,pre} - (1+e)\frac{m_j}{m_i+m_j}[(\boldsymbol{v}_{i,pre} - \boldsymbol{v}_{j,\boldsymbol{p}re}) \cdot \boldsymbol{n}]\boldsymbol{n}, \quad (1)$$

$$\boldsymbol{v}_{j,\boldsymbol{p}ost} = \boldsymbol{v}_{j,\boldsymbol{p}re} + (1+e)\frac{m_i}{m_i+m_j}[(\boldsymbol{v}_{i,pre} - \boldsymbol{v}_{j,\boldsymbol{p}re}) \cdot \boldsymbol{n}]\boldsymbol{n}, \quad (2)$$

where $0 \leq e < 1$ is the coefficient of restitution and $e = -[(\boldsymbol{v}_{i,post} - \boldsymbol{v}_{j,\boldsymbol{p}ost}) \cdot \boldsymbol{n}]/[(\boldsymbol{v}_{i,pre} - \boldsymbol{v}_{j,\boldsymbol{p}re}) \cdot \boldsymbol{n}]$, $\boldsymbol{v}_{\rm pre}$ is the pre-collisional velocity, $\boldsymbol{v}_{\rm post}$ is the post-collisional velocity, $\boldsymbol{n}$ is the unit vector connecting the

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centers of particles i and j.

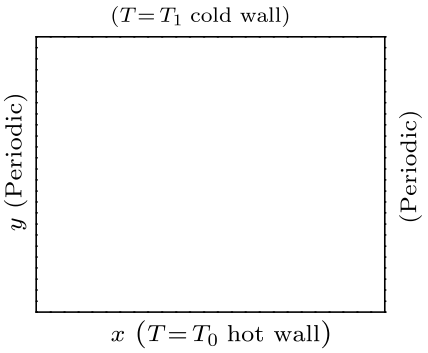

Fig. 1. Simulation domain.

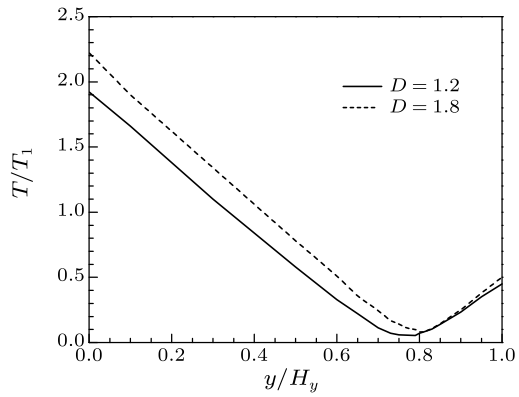

Fig. 2. Dimensionless granular temperature T/T_1 versus y/H_y with $T_0/T_1 = 5$, $e = 0.9$, $\phi_{\rm overall} = 0.1$ ($\phi_{\rm overall}$ is the overall area fraction of the particles).

For the current effort, the particles are assigned in diameters consistent with a power-law size distribution. The diameter of the particles can be expressed as[10−12]

$$d = d_{\max}\left[\frac{N}{n_0}(1 - Y_{N_r})\right]^{-1/D}, \qquad (3)$$

where Y_{Nr} is the occurrence probability of particles that are smaller than d, N is the total number of particles, n_0 is the number of particles with the maximum size $d_{\max}$, and D is the fractal dimension, $1 < D < 2$. The fractal dimension D, as in detail in previous work,[10,11] can be regarded as a measurement of the inhomogeneity of the particle size distribution. A higher D implies a greater dispersion of smaller particles in the system. Hence, the particle size distribution becomes more inhomogeneous as D is increased.

For the purposes of data collection and presentation, the particles are assigned to "families" based on their diameter. Each simulation includes 10 families and 3000 particles. Therefore, the smallest 300 particles are assigned to family 1 and the largest 300 particles are attributed to family 10. Furthermore, the spatial variations of the solid area fraction, the granular temperature and the local size distribution are determined by dividing the simulation domain into strips in the direction parallel to the temperature walls. These data-collection strips are slightly wider than $d_{\rm rms}$, the root-mean-square diameter of particles and $d_{\rm rms} = \sqrt{\frac{\sum_{i=1}^{N} d_i^2}{N}}$. Therefore, the solids fraction of the selected strip is calculated by $\phi_{\rm strip} = \frac{\sum_{i=1}^{N} A_{i,strip}}{A_{\rm strip}}$, where $A_{i,strip}$ is the area of the portion of particle i within the selected strip, $A_{\rm strip}$ is the area of the selected strip.

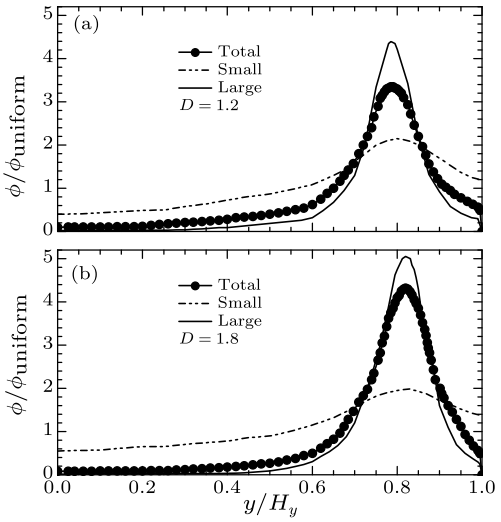

Fig. 3. Behavior of $\phi/\phi_{\rm uniform}$ versus y/H_y with $T_0/T_1 = 5$, $e = 0.9$, $\phi_{\rm overall} = 0.1$.

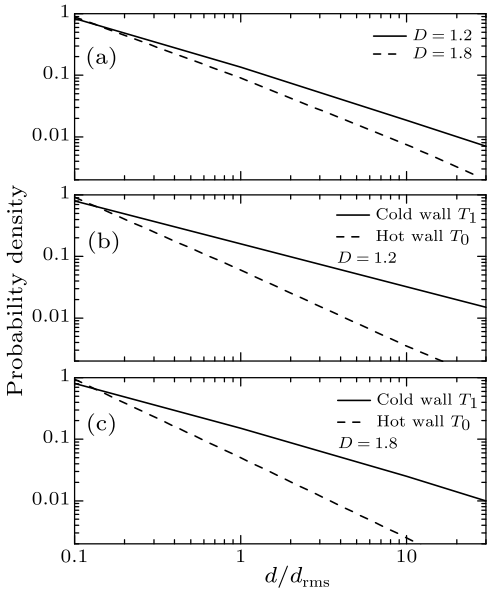

Fig. 4. (a) The overall particle size distribution with $\phi_{\rm overall} = 0.1$. (b) The local particle size distribution near the cold wall and hot wall with $T_0/T_1 = 5$, $e = 0.9$, $\phi_{\rm overall} = 0.1$, $D = 1.2$. (c) The local particle size distribution near the cold wall and hot wall with $T_0/T_1 = 5$, $e = 0.9$, $\phi_{\rm overall} = 0.1$, $D = 1.8$.

The present study is focused on segregation due to a granular temperature gradient. In order to impose a temperature gradient, the temperature of the walls at $y/H_y = 0$ and $y/H_y = 1$ are set to T_0 and

T_1, respectively. After reaching a statistical steady state, the granular temperature profile across the simulation domain was measured. The variation in the dimensionless granular temperature T/T_1 is shown in Fig. 2. Although T/T_1 decreases nearly linearly for $y/H_y < 0.62$ in the system, a nonlinear behavior is observed for $y/H_y \geq 0.62$, with a global minimum of $T/H_y \approx 0.8$. This deviation of the measured granular temperature profile from a linear profile is caused by the inelastic collisions between particles. Each inelastic collision dissipates granular energy which thereby causes the granular temperature to attain values lower than the cold-wall temperature T_1 in the region near the cold wall.

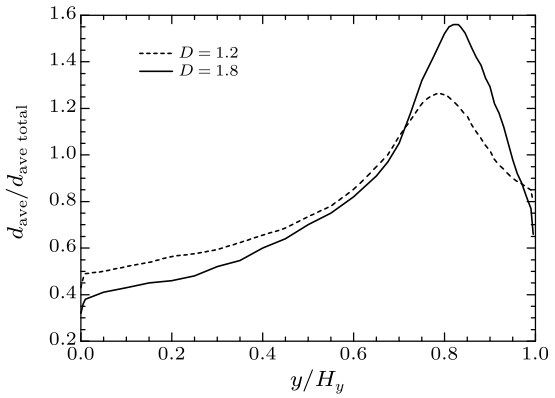

Fig. 5. The scaled mean diameter $d_{\rm ave}/d_{\rm ave,total}$ versus y/H_y with $T_0/T_1 = 5$, $e = 0.9$, $\phi_{\rm overall} = 0.1$.

Now that the spatial variation of the granular temperature has been established, an examination of the corresponding segregation patterns is useful. The segregation can be investigated by comparing the observed solids fractions of each particle family (ϕ) to the solids fractions that would be expected if all particles in a given family are uniformly distributed throughout the interior of the simulation domain ($\phi_{\rm uniform}$). Figure 3 shows the variation of $\phi/\phi_{\rm uiform}$ across the simulation domain for all particles (total: ϕ is equal to the solid fraction of all particles), the family of the smallest particles (small: ϕ is equal to the solid fraction of the smallest particles) and the family of the largest particles (large: ϕ is equal to the solid fraction of the largest particles). The "total" curve in Fig. 3 represents the overall segregation in the system, while the "small" and "large" curves illustrate the species segregation of particles in families 1 and 10, respectively. The total curve in Fig. 3(a) reveals that the particles concentrate near $y/H_y = 0.8$ where the granular temperature is at its minimum. The overall segregation of the mixture is toward regions of lower granular temperature. The smallest and largest particles [the small and large curves in Fig. 3(a), respectively] also concentrate in the low temperature regions. However, species segregation between the smallest and largest particles is observed, in which the concentration of the largest particles is larger than the concentration of the smallest particles at $y/H_y = 0.8$. Therefore, when all particles prefer regions of lower granular temperature, the large particles appear to have a higher affinity for areas of low granular temperature. Figure 3(b) shows that similar trends are observed for the system with $D = 1.8$.

The behavior of systems with continuous size distribution parallels the behavior observed in binary mixtures.[6,8,9,13,14] Nevertheless, one characteristic of the species segregation that is different between binary and continuous-size-distribution systems is of nature of the local size distribution. The local size distribution of a binary mixture will always remain binary in form since only two particle types are present. However, in segregating mixtures with continuous size distribution, the form of the local size distribution is unknown a priori. The overall particle size distributions are shown in Fig. 4(a). Figures 4(b) and 4(c) show the measured local particle size distributions in the regions near the cold wall and hot wall with $D = 1.2$ and $D = 1.8$, respectively. It is found that the local distributions remain essentially power-law distribution even in the presence of species segregation. In addition, the mean diameter of the local distribution is lower near the hot wall and higher near the cold wall. It is worthwhile to note that the quantity is collected throughout the interior of the simulation domain, though the results presented here are limited to the cold-wall and hot-wall regions for the sake of brevity.

Figure 5 gives a more complete view of the variation of the mean deviation of the local distributions throughout the domain. The mean diameter $d_{\rm ave}$ is scaled by the mean diameter for the overall distribution $d_{\rm ave,total}$ (for all 3000 particles). It is found that $d_{\rm ave}$ of local size distribution increases as the granular temperature decreases and the large particles appear to have a higher affinity for areas of low granular temperature.

In our simulations, the granular temperature ratio T_0/T_1 is set to 5. Increasing the granular temperature ratio generally increases both overall and species segregation, but a limiting behavior is observed at extremely large values of T_0/T_1. Figure 6 shows the segregation behavior for $T_0/T_1 = 100$, $T_0/T_1 = 1000$ and $T_0/T_1 = 10000$. Increasing the granular temperature ratio from $T_0/T_1 = 5$ to $T_0/T_1 = 100$ results in a corresponding increase in the peak $\phi/\phi_{\rm uiform}$ value for the largest particle family from $\phi/\phi_{\rm uiform} \approx 5$ to $\phi/\phi_{\rm uiform} \approx 9.8$. The next increase in the granular temperature ratio from $T_0/T_1 = 100$ to $T_0/T_1 = 1000$ results in less change, with the peak $\phi/\phi_{\rm uiform}$ value for the largest particle family increasing slightly from $\phi/\phi_{\rm uiform} \approx 9.8$ to $\phi/\phi_{\rm uiform} \approx 12$. Increasing T_0/T_1 again, this time to $T_0/T_1 = 10000$, causes almost no change in $\phi/\phi_{\rm uiform}$ for the largest particle family. The value of $\phi/\phi_{\rm uiform}$ for all particles (total curves) follows the same trend as T_0/T_1 increases. The peak in $\phi/\phi_{\rm uiform}$ for the total curve generally moves closer to the cold wall as T_0/T_1 is increased, but does reach a limiting value at extremely large values of T_0/T_1 because of space restrictions (i.e., particles cannot pene-

trate the wall at $y/H = 1$). The value of $\phi/\phi_{\text{uiform}}$ for smallest particle family changes little as T_0/T_1 increases. The $\phi/\phi_{\text{uiform}}$ curve for the smallest particles decreases near $y/H_y = 0.95$ as T_0/T_1 is increased. This decrease in $\phi/\phi_{\text{uiform}}$ near $y/H_y = 0.95$ is caused by the presence of larger particles in a nearly packed state which leaves less room for the smaller particles to occupy. Moreover, in Fig. 6(c), a peak in $\phi/\phi_{\text{uiform}}$ for the smallest particles is observed at $y/H_y \approx 1$. This peak is a side effect of the extremely high value of $T_0/T_1 = 10000$. The large granular temperature ratio causes rapid segregation and essentially traps small particles between the wall at $y/H_y = 1$ and the low temperature band of large particles at $y/H_y = 0.95$. It is worth noting that binary mixtures may attain a state of complete species segregation, in which the two species are essentially completely separated. Complete segregation generally requires large granular temperature gradients or large size and/or mass differences between the species.[15] However, complete segregation has not been observed in any of the simulations performed for continuous size distribution.

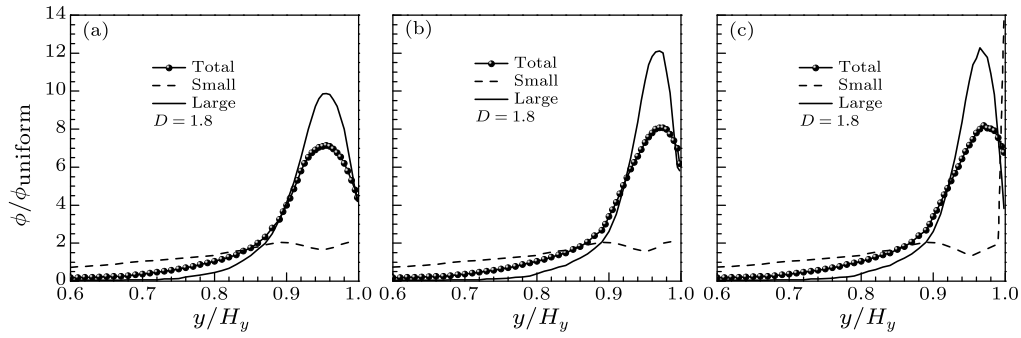

Fig. 6. Characteristics of $\phi/\phi_{\text{uniform}}$ versus y/H_y with $T_0/T_1 = 5$, $e = 0.9$, $\phi_{\text{overall}} = 0.1$: (a) $T_0/T_1 = 100$, (b) $T_0/T_1 = 1000$ and (c) $T_0/T_1 = 10000$.

In summary, numerical simulations of a granular mixture with a power-law size distribution are completed to elucidate the segregation behavior in the presence of a granular temperature gradient. In general, the particles of all sizes move towards the areas of the lowest granular temperature. The larger particles have a higher affinity for the low granular temperature regions and concentrate more in these regions than their smaller counterparts. The local particle size distribution tends to remain of the same type as the overall particle size distribution. The mean diameter of the local particle size distribution increases as the granular temperature decreases. In addition, the overall and species segregation both are enhanced by increasing the granular temperature gradient T_0/T_1. However, complete segregation is not observed even at extremely high values of T_0/T_1. Much work remains to be performed on continuum models before a complete theory for continuous size distributions is realizable.

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