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Phantom Energy Accretion by a Stringy Charged Black Hole *

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We investigate the dynamical behavior of phantom energy near a stringy magnetically charged black hole. For this purpose, we derive equations of motion for steady-state spherically symmetric flow of phantom energy onto the stringy magnetically charged black hole. It is found that phantom energy accreting onto a black hole decreases its mass. Further, the location of the critical points of accretion is explored, which yields a mass to charge ratio. This ratio implies that accretion process cannot transform a black hole into an extremal black hole or a naked singularity, hence cosmic censorship hypothesis remains valid here.

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Astronomical observations of our universe have provided evidence for the existence of an unusual type of matter known as dark energy (DE), which governs the expansion of the universe.[1−4] It is estimated that two-thirds of our universe is made up of DE, which has a large negative pressure and can drive the accelerated expansion of the universe.[5] It exhibits some unusual properties such as a negative value of the equation of state (EoS) parameter and violation of energy conditions.[6] Numerous models have been proposed as candidates of DE such as vacuum energy, quintessence, phantom and Chaplygin gas. Vacuum energy (cosmological constant) is the simplest form of DE for which the EoS parameter is $\omega = -1$. The quintessence and phantom are the forms of DE for which $\omega > -1$ and $\omega < -1$, respectively.[7−9] Saridakis[10] discussed the theoretical limits on the EoS parameter in phantom cosmology, and has also investigated phantom evolution using power-law potentials.[11] Leon and Saridakis[12] used the phantom dark energy model to explore the acceleration and coincidence problem in cosmology. Phantom energy violates the dominant energy condition.

A massive object surrounded by a matter can capture particles of matter that pass within a certain distance from the massive object. This phenomena is termed as accretion of matter by massive objects. Bondi[13] originally formulated the problem of matter accretion by the compact objects in Newtonian gravity. Michel[14] is the pioneer who studied accretion of gas onto the Schwarzschild black hole (BH) in relativistic physics. Sun[15] discussed phantom energy accretion onto a BH in the cyclic universe. Babichev *et al.*[16] have shown that BH mass diminishes due to phantom accretion. Jamil *et al.*[17] have explored the effects of phantom energy accretion onto a charged BH in 4D. They pointed out that if the mass of a BH becomes smaller (due to accretion of phantom energy) than its charge, then cosmic censorship hypothesis is violated. The same conclusion was deduced by Babichev *et al.*[18] in studying the phantom accretion onto charged BH with generalized linear EoS and Chaplygin gas EoS. In recent papers,[19,20] we have studied phantom accretion onto Schwarzschild de-Sitter and 5D charged BHs.

Stringy BHs have been a subject of interest for the last few years, due to the fact that string theory is a clearly defined theory of quantum gravity. General relativity with some new matter fields, such as the dilaton and the axion, resembles the low energy effective theory. Kar[21] and Dasgupta *et al.*[22] studied energy conditions and the kinematics of the geodesic flow of charged stringy BHs. Sharif[23] investigated the structure of force and potential for stringy BHs by using the pseudo-Newtonian formulation. Sharif and Waheed[24] discussed the re-scaling of energy fot this BH by using the approximate symmetry approach. Radinschi and Ciobanu[25] explored the energy momentum distribution for charged BHs. Motivated by these studies, we investigate phantom accretion by string magnetically charged BHs.

In this study, we follow the formulation of Michel[17] to discuss phantom accretion. It is found that phantom accretion cannot transform a BH into a naked singularity or an extremal BH, in contrast to the Reissner–Nordström (RN) BH. Gravitational units (i.e., the gravitational constant $G = 1$ and speed of light in vacuum $c = 1$) are used.

The stringy magnetically charged BH solution is

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given by[26]

$$ds^2 = \frac{1-\frac{m}{r}}{1-\frac{Q^2}{mr}}dt^2 - \frac{1}{(1-\frac{m}{r})(1-\frac{Q^2}{mr})}dr^2 - r^2(d\theta^2+\sin^2\theta d\phi^2), \quad (1)$$

where m and Q are the mass and charge of the BH. For $Q=0$ and $m=2M$, we obtain the Schwarzschild solution. The BH horizons can be found by solving $g^{11}=0$, which leads to

$$r_\pm = \frac{(m^2+Q^2) \pm \sqrt{(m^2-Q^2)^2}}{2m}, \quad (2)$$

where $r_\pm$ imply that there are two horizons $r_+ = m$ and $r_- = \frac{Q^2}{m}$ such that $r_+ > r_-$. For $m^2 = Q^2$. We obtain $r_+ = r_- \equiv m$, which is the case of an extremal charged BH. Unlike RN BHs for $m^2 < Q^2$, both horizons exist and one cannot obtain a naked singularity at $r=0$.

The energy-momentum tensor for perfect fluid reads

$$T_{\mu\nu} = (\rho+p)u_\mu u_\nu - p g_{\mu\nu}, \quad (3)$$

where ρ is the energy density, p is the pressure and $u^\mu = (u^t, u^r, 0, 0)$ is the four-vector velocity. We mention here that u^μ satisfies the normalization condition, i.e., $u^\mu u_\mu = 1$. Conservation of the energy-momentum tensor yields

$$\frac{r^2 u}{1-\frac{Q^2}{mr}}(\rho+p)\Big((1-\frac{Q^2}{mr})(1-\frac{m}{r})+u^2\Big)^{1/2} = D_0, \quad (4)$$

where D_0 is an integration constant and $u^r = u < 0$ for inward flow of a phantom towards the BH .

The energy flux equation can be derived by projecting the energy-momentum conservation law on the four-velocity, i.e., $u_\mu \nabla_\nu T^{\mu\nu} = 0$ for which Eq. (3) leads to

$$\frac{r^2 u}{1-\frac{Q^2}{mr}}\exp(n) = -D_1, \quad (5)$$

where $D_1 > 0$ is another integration constant, which is related to the energy flux and $n = \int_{\rho_\infty}^{\rho_h} \frac{d\rho'}{\rho'+p(\rho')}$. Also, ρ_h and ρ_∞ are densities of the phantom energy at horizon and infinity. From Eqs. (4) and (5), we have

$$(\rho+p)\Big((1-\frac{Q^2}{mr})(1-\frac{m}{r})+u^2\Big)^{1/2}\exp(-n) = D_2, \quad (6)$$

where $D_2 = -\frac{D_0}{D_1} = \rho_\infty + p\,(\rho_\infty)$.

The rate of change of BH mass due to phantom energy accretion is[18]

$$\dot{M} = -4\pi r^2 T^r{}_t. \quad (7)$$

Using Eqs. (4)–(6) in the above equation, we obtain

$$\dot{M} = 4\pi D_1(\rho_\infty + p_\infty), \quad (8)$$

which implies that the mass of a BH decreases if $(\rho_\infty + p_\infty) < 0$. Thus the accretion of phantom energy onto a BH leads to decrease of the mass of the BH. The phantom energy accretion only diminishes mass and does not affect the charge of the BH. That is why an RN BH is converted into a naked singularity by the phantom accretion and CCH is violated. However, the critical accretion process mentioned below implies that CCH remains valid in this case. Since all p and ρ, violating dominant energy condition, must satisfy this equation, it holds in general. We would like to mention here that the above relation is same for all 4D spherically symmetric BHs.

Here we locate such points at which flow speed is equal to the speed of sound during accretion. The fluid falls onto the BH with monotonically increasing velocity along the particle trajectories. We discuss the critical accretion. The conservation of mass flux, $\nabla_\mu J^\mu = 0$, gives

$$\frac{\rho u r^2}{1-\frac{Q^2}{mr}} = k, \quad (9)$$

where k is an integration constant. It is obvious that $k<0$ as $u<0$ and all the other quantities are positive. Using Eqs. (4) and (9), we obtain

$$\Big(\frac{\rho+p}{\rho}\Big)^2\Big((1-\frac{Q^2}{mr})(1-\frac{m}{r})+u^2\Big) = k_1, \quad (10)$$

where $k_1 = (\frac{C_0}{k})^2$ is a positive constant. Differentiating Eqs. (9) and (10) and eliminating $d\rho$, we have

$$\frac{dr}{r}\Big(2V^2 - \frac{\frac{1}{2}(\frac{M}{r}-\frac{2Q^2}{r^2}+\frac{Q^2}{mr})}{(1-\frac{Q^2}{mr})(1-\frac{m}{r})+u^2}\Big) + \frac{du}{u}\Big(V^2 - \frac{u^2}{(1-\frac{Q^2}{mr})(1-\frac{m}{r})+u^2}\Big) = 0, \quad (11)$$

where $V^2 = \frac{dln(\rho+p)}{dln\rho} - 1$. The critical points are found by taking both the factors inside the brackets equal to zero. Thus we obtain

$${u_c}^2 = \frac{1}{4mr_c^2}r_c(m^2+Q^2) - 2mQ^2, \quad (12)$$

$${V_c}^2 = \frac{r_c(m^2+Q^2)-2mQ^2}{5mr^2 - 3r_c(Q^2+m^2)+2Q^2m}. \quad (13)$$

We find that the solutions of the above equations are obtained if ${u_c}^2 > 0$ and ${V_c}^2 > 0$ implying that

$$r_c(m^2+Q^2) - 2mQ^2 > 0, \quad 5mr^2 - 3r_c(Q^2+m^2)+2Q^2m > 0. \quad (14)$$

The subscript c is used to represent a quantity at a point where speed of flow is equal to the speed of

sound, such a point is called a critical point. The second one in Eq. (14) has the solution

$$r_{c\pm} = 3(m^2+Q^2) \pm \sqrt{(9m^4 - 22m^2Q^2 + 9Q^4)}, \quad (15)$$

which will be real if

$$\frac{m^2}{Q^2} \geq \frac{1}{9}(11 + 2\sqrt{10}) \approx 1.925. \quad (16)$$

The location of the critical points near the BH can be determined by the roots $r_{c\pm}$. For the solution about critical point, we insert the value of $r_{c\pm}$ in the first one of Eq. (14) and obtain a unique inequality

$$\frac{m^2}{Q^2} > \frac{1}{2}(3+\sqrt{5}) \approx 2.618. \quad (17)$$

Since $r_{c\pm}$ remains real for the above ratio, accretion is possible through $r_{c\pm}$ as long as the above inequality is satisfied.

In summary, there always exist two horizons for stringy magnetically charged BH independent of the m^2-to-Q^2 ratio of BH. In other words, whatever the mass-to-charge ratio would be, a stringy magnetically charged BH cannot be converted into a naked singularity. We would like to mention that for $\frac{Q^2}{m^2} > 1$, the horizons for the RN BH disappear but there does not exist such a ratio in the present case for which horizons disappear. The critical accretion analysis implies that corresponding to two horizons, there exist two values of $r_{c\pm}$, which can play the role of critical points if the mass and charge of BH satisfy $\frac{m^2}{Q^2} > \frac{1}{2}(3+\sqrt{5}) \approx 2.618$. It is concluded that phantom accretion decreases the mass of a BH and converts it to an extremal BH if $m^2 = Q^2$. However, during the accretion, we have $\frac{m^2}{Q^2} > \frac{1}{2}(3+\sqrt{5}) \approx 2.618$. Thus unlike RN BHs, stringy magnetically charged BHs cannot be transformed to an extremal charged BH or a naked singularity and CCH remains valid here.

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