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Slow-Light Propagation in a Tapered Dielectric Periodic Waveguide over Broad Frequency Range *

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A tapered waveguide composed of a one-dimensional periodic arrangement of dielectric material is proposed for light trapping. The equifrequency contours (EFC) of silicon-air multilayer photonic crystals within the first band-gap region are first studied. A zero-group-velocity at the first Brillouin zone boundary along the grating vector is predicted. The propagation constants and eigenfrequencies of the first-order guiding modes are numerically investigated for photonic crystal waveguide structures with a finite thickness. Different frequency components of the guiding modes are found to slow and stop at different thicknesses inside such a tapered waveguide structure. In addition, the time-evolution of a femto-second pulse propagating in the tapered waveguide is also demonstrated.

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Optical devices capable of slowing down, stopping or capturing light have attracted a great deal of attention during the past several years because of their profound implications for classical and quantum information processing.[1−8] To date, several methods such as electromagnetically induced transparency,[1] quantum-dot semiconductor optical amplifiers,[4] photonic crystals,[5] coupled-resonator optical waveguide[6] and surface plasmon polaritons in metallic waveguides[7−9] have been suggested to realize slow light. However, most of these aforementioned methods have involved extremely low temperatures, have been extremely costly and have only worked with one specific frequency of light. Recently, Tsakmakidis *et al.*[10,11] demonstrated theoretically that an axially varying heterostructure with a metamaterial core of negative refractive index can be used to efficiently and coherently bring light to a complete standstill. In this way, different frequencies of a white light could be stopped at different stages down the taper, thereby creating a 'trapped rainbow'. Moreover, several novel waveguide structures consisting of negative effective index or negative epsilon material can also be used to trap light.[12−14]

In this Letter, we propose that a tapered waveguide structure composed of a periodic arrangement of dielectric material could be used to capture light. We first calculate the band structure at normal incidence and the equifrequency contours (EFCs) of one-dimensional silicon-air multilayer photonic crystals. The propagation constants and eigenfrequencies of the first-order guiding modes in a photonic crystal waveguide with a finite-thick are then analyzed numerically. A zero group velocity at the first Brillouin zone (BZ) boundary along the grating vector is predicted and a wheel-shaped power flow is formed simultaneously. Different frequency components of the guiding modes are found to stop at correspondingly different thicknesses inside such a tapered waveguide structure. Finally, we also demonstrate the time-evolution of a femto-second pulse propagating in the tapered waveguide.

We use a commercial FEM software package (COMSOL Multiphysics) to numerically calculate light trapping properties of the proposed photonic crystal waveguide structures. In the simulations, only transverse magnetic (TM) polarization is considered (all the results for the case of transverse electric polarization can be obtained in a similar way).

In one-dimensional (1-D) photonic crystal, Bloch wave is the generic solution of Maxwell's equations. For TM polarization, the exact Bloch wave dispersion relation in this structure is given by[15,16]

$$\begin{aligned}&\cos[k_x(a_1+a_2)]\\&=\cos(k_1a_1)\cos(k_2a_2)-\left(\frac{\varepsilon_d^2k_1^2+k_2^2}{2\varepsilon_dk_1k_2}\right)\\&\times\sin(k_1a_1)\sin(k_2a_2),\end{aligned}\tag{1}$$

where $k_1=\sqrt{k_0^2-k_z^2}$, $k_2=\sqrt{\varepsilon_dk_0^2-k_z^2}$, a_1 is the air gap width, a_2 is the dielectric layer (silicon) width, k_x and k_z are the x- and z-components of free space wave vector k_0, respectively. The schematic of the considered 1-D periodic structure and the coordinate are shown in the inset of Fig. 1(a). The width of the silicon ($\varepsilon=12$) layer is $a_2=160\,\text{nm}$ and the width of the air ($\varepsilon=1$) layer is $a_1=200\,\text{nm}$. By using Eq. (1), we first calculate the band structure for nor-

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mal incidence (the wave vector is along the x-axis and $k_z = 0$). It is clearly seen from Fig. 1(a) that a band-gap ranging from 1285 nm to 2117 nm is opened for normal incidence. Note that in 1-D photonic crystals this band-gap is only a pseudo directional gap. When the light is inclined incident, i.e., the z-component of wave vector is non-zero ($k_z \neq 0$), some guiding modes can possibly propagate at the frequency within the directional band-gap originally opened for normal incidence.

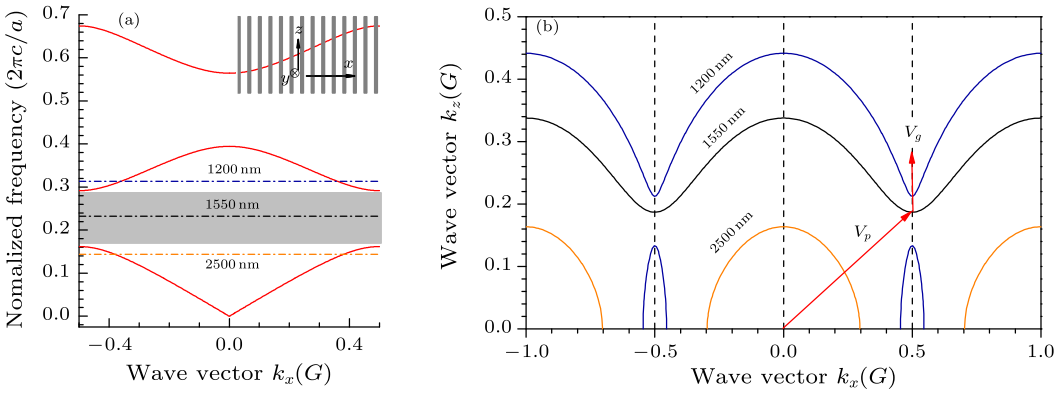

Fig. 1. (a) Band structure of the silicon-air multilayer one-dimensional photonic crystals. The shadow region represents the first band gap; (b) the blue curve is the dispersion relation at the operating wavelength of 1200 nm. The black sine curve is the dispersion relation of a silicon-air multilayer at the operating wavelength of 1550 nm for TM polarization. The lower dispersive yellow curve is the dispersion relation at the operating wavelength of 2500 nm. The upper right inset of the figure show the geometry of the silicon-air periodic dielectric multilayer. The arrows mark the group velocity v_g and the phase velocity v_p at $k_x = G/2$.

Now we shall study the EFC of the considered 1-D periodic structure, which illustrates the relation between k_x and k_z. At the same time, in the plot of EFC we can obtain one of the most important parameters, the direction of group velocity V_G, which should be perpendicular to the EFC curve and points toward larger frequencies.[17,18] To achieve the slow light effect in one specified direction, the component of the group velocity along this direction is required to be almost zero. As a first step, three different wavelengths are chosen to calculate the EFC. Note that in Fig. 1(a) these three wavelengths correspond to the value locating at the first energy band (2500 nm), within the directional band-gap (1550 nm) and at the second energy band (1200 nm). As shown in Fig. 1(b), for the wavelength 2500 nm only one discrete curve exists and for the wavelength 1550 nm only a continuous curve appears. While for the wavelength 1200 nm both continuous and discrete curves are observed. Immediately, we can see that at the edge of first BZ the group velocity is almost along the z direction and the x-component is almost zero for the wavelengths of 1200 nm and 1550 nm, which provides us a possibility to achieve the slow light effect. Furthermore, according to our calculations for the wavelength located within the directional band-gap there is always only one cosine shaped EFC curve (data not shown here). Also, since the EFC curve for the wavelength located at the second energy band is not a single valued function, in the following discussions we will restrict ourselves to design a tapered dielectric periodic waveguide operating within the directional band-gap to realize the slow light effect.

From the above discussions we have considered different dispersion relation profiles of an infinite silicon-air multilayer one-dimensional photonic crystal and select a certain dispersion profile. A slab periodic waveguide consisting of a finite-thick dielectric-air multilayer core surrounded by conventional homogeneous medium (air in present paper) is investigated under the condition that the dispersion relation in the core layer is the sine curve. Eigenfrequencies versus propagation constants of first-order guiding modes at different thicknesses of core are obtained and illustrated in Fig. 2(a) by using the eigenfrequency analysis module in the in-plane waves module of COMSOL. The multilayer we choose is the same as the dielectric multilayer structure we calculate in Fig. 1. The shadow region represents the directional band-gap (for normal incidence) of this periodic waveguide, in which the curves are continuous and have a solution at the edge of first BZ ($k_x = G/2$, $G = 2\pi/a$).

As shown in Fig. 2(a), the direction of the group velocity $v_g = \partial\omega/\partial k_x$ is changed in different zones. In the first BZ, the mode is a forward mode. In the second BZ, the mode is a backward mode since directions of the energy flow along the guide and the phase velocity are opposite. At a special point $k_x = G/2 = \pi/(a_1 + a_2)$, the group velocity becomes zero. The propagation constants of the waveguide structure is symmetric with respect to the line $k_x = G/2$. The dispersion relation of slab waveg-

uide is consistent with the dispersion character of the core. Obviously, propagation constant we interested is around the edges of the first Brillouin zone. From the energy flow direction arrowed in Fig. 2(b), it is clear to see that the wheel-shaped power flow induces the stopping of light.

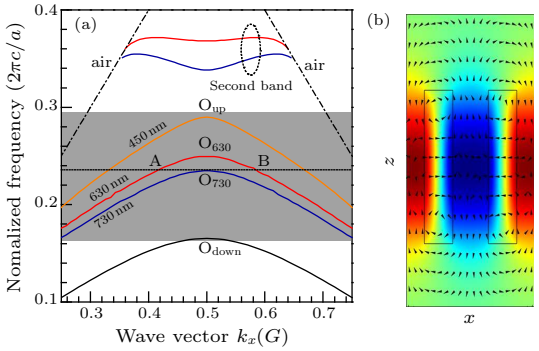

Fig. 2. (a) Frequencies versus the propagation constants of the finite-thick photonic crystal waveguide structures. (b) The z component of the magnetic field distribution of this waveguide for propagation constant $k_x = G/2$.

Next we investigate light propagation along a linearly tapered waveguide. In Fig. 2(a), we can see that the blue curve represents the frequencies-propagation constants curve at waveguide thickness 730 nm. When the thickness of the waveguide reduces to 630 nm (red curve), the profile of the curve in this case is similar to 730 nm except that eigenfrequencies become larger. The dotted line in Fig. 2(a) represents the waveguide eigenmodes at about 1550 nm. The waveguide with the core thickness of 730 nm is cut off at operating wavelengths above the black dotted line. From the dot line, we can learn that at a certain wavelength, the group velocity decreases as the thickness of the waveguide increase within the first Brillouin zone. As shown in Fig. 3(a), in the tapered waveguide, the width of this narrow end is 630 nm. At the narrow end, we excite eigenmode at point A in Fig. 2(a). As the core thickness increases, eigenmode at point A gradually change to eigenmode at point O_{730}. When the phase velocity reaches the edges of the first Brillouin zone $k_x = G/2$, the group velocity along the tapered waveguide reached zero. Therefore, light stopped at the certain thickness 730 nm with the eigenmode at point O_{730}. Increasing the waveguide thickness further will lead to the operating waveguide 1550 nm below the cutoff wavelength. Light cannot propagate in higher thickness than 730 nm and stopped at this thickness.

Different operating wavelengths correspond to different stopping thicknesses. When the wavelength changes to 1600 nm, as shown in Fig. 3(b), light is stopped at the position with waveguide thickness 780 nm. The higher wavelength light is trapped at the location with larger waveguide thickness. When the incident light is white light, the spatial separation of different wavelength light will construct a 'trapped rainbow'. This approach is different from previous 'trapped rainbow' designs caused by the presence of the effective negative index component in the waveguide. The special characters at the edge of the first BZ is the main physic origin which leads to the similar 'trapped rainbow' phenomenon. At the edge of the first BZ, the wheel-shaped power flow induce the stopping of light. The wheel-shaped power flow is caused by the energy flow in the core and the energy flow with opposite direction in the cladding.[19] The dimension of a light wheel along the x direction is the reciprocal vector of G and the dimension along the z direction is with respective to the wavelength. Therefore the longer wavelength light is trapped at the location with larger waveguide thickness.

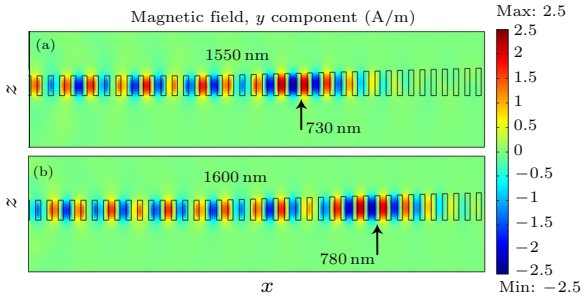

Fig. 3. Light propagation and trapping in the linearly tapered waveguide with a silicon-air periodic dielectric multilayer core and air cladding. (a) Light propagation in this structure with operating wavelength 1550 nm. The waveguide thickness at the location of light trapping is 730 nm. (b) Light propagation in this structure with operating wavelength 1600 nm. The thickness at the location of light trapping is 780 nm. All the geometry parameters of the multilayer core are identical to those in Fig. 2.

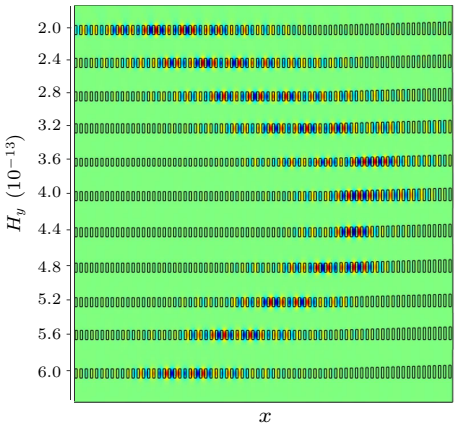

Fig. 4. A pulse propagates in the tapered waveguide with pulse duration 1×10^{-13} s. Wavelength is 1550 nm. Time interval is 0.4×10^{-13} s. The pulse is compressed at the trapping location and propagates backward after staying for a while.

Now we discuss the leaky properties of wave propagation in the above tapered waveguide. Along the x direction, due to the variation of the core thickness,

the properties of the guided eigenmode in the tapered waveguide is changed and then guided mode become leaky. However, if we use a very long adiabatically tapering structure, the leak of the tapered waveguide can decrease.

Additionally, the amplitude of magnetic field components at the trapped location is about 2 times of the values at other location with forward propagation mode due to the fact that slower light has higher energy intensity. It is a useful character for nonlinear and strong light-matter interaction.

We should also note that it is not easy to realize the light trapping forever since the backward mode will excite after the pulse is trapped at the trapping location. We study the propagation of a pulse with pulse duration 1×10^{-13} s. As shown in Fig. 4, the pulse is slowed down and compressed along the tapered waveguide. At the stopping location, the pulse stay here for a while (the order of 1×10^{-13} s) and gradually propagate backward.

In conclusion, we have derived the dispersion relations of a periodic dielectric slab waveguide. The energy flow along the direction of propagation is zero at the edge of the first BZ. We also study the dependence of the thicknesses of the trapping position on the excitation wavelengths in this structure and design an appropriate approach to slow and trap light in a broad frequency range. Our simulation shows that the relation between the thicknesses and wavelength is similar to that of the 'trapped rainbow' effect shown in Ref. [9] and the Bragg structure can realize this effect instead of the negative effective index component. Lastly, we study the light pulse staying time of this structure. This easy-fabricated nanostructure is designed at around 1550 nm and can be easily scaled down to optical range or other wavelength ranges.

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