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# An Interacting Two-Fluid Scenario for Dark Energy in an FRW Universe

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*We study the evolution of the dark energy parameter within the scope of a spatially homogeneous and isotropic Friedmann–Robertson–Walker (FRW) model filled with barotropic fluid and dark energy. To obtain the deterministic solution we choose the scale factor $a(t) = \sqrt{te^t}$, which yields a time-dependent deceleration parameter (DP). In doing so, we consider the case minimally coupled with dark energy to the perfect fluid as well as direct interaction with it.*

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Observations of distant Supernovae (SNe Ia),[1−6] fluctuation by cosmic microwave background radiation (CMBR),[7,8] the large scale structure (LSS),[9,10] the sloan digital sky survey (SDSS),[11,12] the Wilkinson microwave anisotropy probe (WMAP)[13] and the Chandra x-ray observatory[14] by means of ground and altitudinal experiments have shown that our Universe is spatially flat and expanding with acceleration. This fact is in agreement with the theory if one assumes that the Universe is basically filled with so-called dark energy. The measurement of photometric distances to the cosmological Supernova, supported by a number of independent arguments, in particular by the observational data on the angular temporal fluctuations by CMBR, shows that the lion's share of the energy density of matter belongs to non-baryonic matter. This form of matter cannot be detected in a laboratory and does not interact with electromagnetic radiation. Given the fact that almost three-fourths of the energy density of the Universe originated from dark energy and plays crucial role in the accelerated mode of expansion of the Universe, there are a large number of models capable of describing this dark energy.

Dark energy models with higher derivative terms were constructed by Zhang and Liu.[15] The cosmological evolution of a two-field dilaton model of dark energy was investigated by Liang *et al.*[16] Viscous dark energy models with variable $G$ and $\Lambda$ were studied by Arbab.[17] The modified Chaplygin gas with interaction between holographic dark energy and dark matter was investigated in Ref. [18]. The tachyon cosmology in interacting and non-interacting cases in a non-flat FRW Universe was studied in Ref. [19]. In this Letter, we study the evolution of the dark energy parameter within the framework of a FRW cosmological model filled with two fluids. In doing so, we consider both interacting and non-interacting cases.

We consider the spatially homogeneous and isotropic Friedmann–Robertson–Walker (FRW) metric as

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \tag{1}$$

where $a(t)$ is the scale factor, the curvature constants $k$ are $-1$, 0, $+1$ for open, flat and closed models of the universe, respectively, and $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$.

The Einstein field equations (with $8\pi G = 1$ and $c = 1$) read

$$R_i^j - \frac{1}{2}R\delta_i^j = -T_i^j, \tag{2}$$

where the symbols have their usual meaning and $T_i^j$ is the two fluid energy-momentum tensor consisting of a dark field and barotropic fluid.

In a co-moving coordinate system, Einstein's field Eq. (2) for the line element (1) leads to

$$p_{\rm tot} = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right), \tag{3}$$

$$\rho_{\rm tot} = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right), \tag{4}$$

where $p_{\rm tot} = p_m + p_D$ and $\rho_{tot} = \rho_m + \rho_D$. Here, $p_m$ and $\rho_m$ are pressure and energy density of barotropic fluid and $p_D$ & $\rho_D$ are pressure and energy density of dark fluid respectively.

The Bianchi identity $G_{ij}^{;j} = 0$ leads to $T_{ij}^{;j} = 0$ which yields

$$\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}(\rho_{tot} + p_{tot}) = 0. \tag{5}$$

The EoS of the barotropic fluid and dark field are given by

$$\omega_m = \frac{p_m}{\rho_m}, \tag{6}$$

$$\omega_D = \frac{p_D}{\rho_D}, \tag{7}$$

respectively. First, we consider that two fluids do not interact with each other. Therefore, the general

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form of conservation equation (5) leads us to writing the conservation equation for the dark and barotropic fluid separately as,

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = 0, \tag{8}$$

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}(\rho_D + p_D) = 0. \tag{9}$$

Integration of Eq. (5) leads to

$$\rho_m = \rho_0 a^{-3(1+\omega_m)}, \tag{10}$$

where $\rho_0$ is an integrating constant. By using Eq. (10) in Eqs. (3) and (4), we first obtain the $\rho_D$ and $p_D$ in term of scale factor $a(t)$

$$\rho_D = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0 a^{-3(1+\omega_m)}. \tag{11}$$

$$p_D = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0\omega_m a^{-3(1+\omega_m)}. \tag{12}$$

Now we take the following *ansatz* for the scale factor, where the increase in terms of time evolution is

$$a(t) = \sqrt{te^t}. \tag{13}$$

The motivation to choose such a scale factor is sue to the fact that the universe is in accelerated expansion at present and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Thus, in general, the DP is not a constant but time variable. By the above choice of scale factor yields a time-dependent DP.

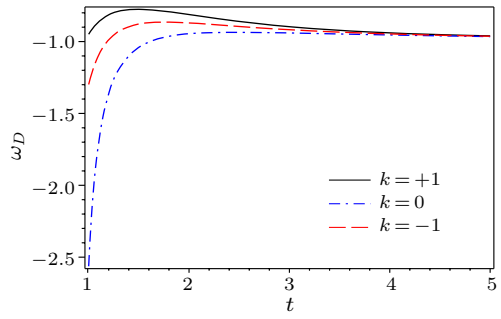



**Fig. 1.** The plot of EoS parameter $\omega_D$ vs $t$ for $\rho_0 = 10$ and $\omega_m = 0.5$.

By using this scale factor in Eqs. (11) and (12), the $\rho_D$ and $p_D$ are obtained as

$$\rho_D = 3\left[\frac{(1+t)^2}{4t^2} + \frac{k}{te^t}\right] - \rho_0(te^t)^{-\frac{3}{2}(1+\omega_m)}, \tag{14}$$

$$p_D = -\left[\frac{3(1+t)^2 - 4}{4t^2} + \frac{k}{te^t} + \rho_0\omega_m(te^t)^{-\frac{3}{2}(1+\omega_m)}\right], \tag{15}$$

respectively. By using Eqs. (14) and (15) in Eq. (7), we find the equation of state of dark field in term of time as

$$\omega_D = -\left[\frac{\frac{3(1+t)^2-4}{4t^2} + \frac{k}{te^t} + \rho_0\omega_m(te^t)^{-\frac{3}{2}(1+\omega_m)}}{3\left[\frac{(1+t)^2}{4t^2} + \frac{k}{te^t}\right] - \rho_0(te^t)^{-\frac{3}{2}(1+\omega_m)}}\right]. \tag{16}$$

The behavior of EoS for DE in term of cosmic time $t$ is shown in Fig. 1. It is observed that although for open, closed and flat universes, the EoS parameter is an increasing function of time, the rapidity of its growth at the early stage depends on the type the universe. Later on it tends to the same constant value independent of the types of the universe.

The expressions for the matter-energy density $\Omega_m$ and dark-energy density $\Omega_D$ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{4t^2}{3(1+t)^2}\rho_0(te^t)^{-\frac{3}{2}(1+\omega_m)}, \tag{17}$$

$$\Omega_D = \frac{\rho_D}{3H^2} = 1 + \frac{4kt}{3(1+t)^2e^t} - \frac{4t^2}{3(1+t)^2} \times \rho_0(te^t)^{-\frac{3}{2}(1+\omega_m)}, \tag{18}$$

respectively. Equations (17) and (18) reduce to

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{4kt}{3(1+t)^2e^t}. \tag{19}$$

From the right-hand side of Eq. (19) it is clear that in flat universe ($k = 0$), $\Omega = 1$ and in open universe ($k = -1$), $\Omega < 1$ and in a closed universe ($k = +1$), $\Omega > 1$. However, at late time we see for all flat, open and closed universes $\Omega \to 1$. This result is compatible with the observational results. Since our model predicts a flat universe for large times and the present-day universe is very closed to flat, the derived model is also compatible with the observational results. The variation of density parameter with cosmic time has been shown in Fig. 2.

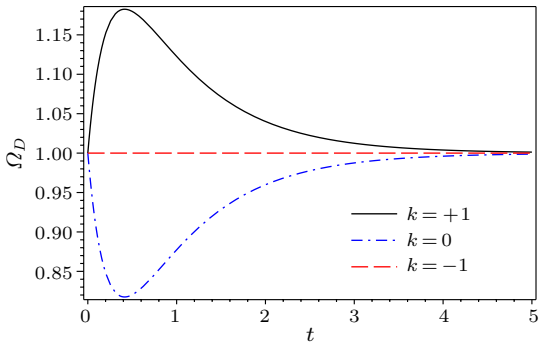



**Fig. 2.** The plot of density parameter $\Omega$ vs $t$ for $n = 3$.

We define the deceleration parameter $q$ as usual, i.e.

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}. \tag{20}$$

Using Eqs. (3) and (4), we may rewrite as Eq. (20) as

$$q = \frac{1}{6H^2}\left[\rho_m(1+3\omega_m) + \rho_D(1+3\omega_D)\right]. \tag{21}$$

On the other hand, using Eq. (13) into Eq. (20), we find

$$q = \frac{2}{(1+t)^2} - 1. \tag{22}$$

From Eq. (22), we observe that $q > 0$ for $t < 0.41$ and $q < 0$ for $t > 0.41$. This behavior of $q$ is clearly depicted in Fig. 3.

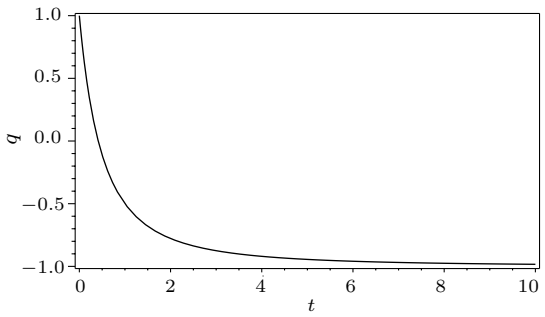



**Fig. 3.** The plot of deceleration parameter $q$ vs $t$.

A convenient method to describe models close to $\Lambda$ CDM is based on the cosmic jerk parameter $j$, a dimensionless third derivative of the scale factor with respect to the cosmic time.[20–23] A deceleration-to-acceleration transition occurs for models with a positive value of $j_0$ and negative $q_0$. Flat $\Lambda$ CDM models have a constant jerk $j = 1$. The jerk parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a}, \tag{23}$$

and in terms of the scale factor to cosmic time

$$j(t) = \frac{(a^2H^2)''}{2H^2}, \tag{24}$$

where the 'dots' and 'primes' denote derivatives with respect to cosmic time and scale factor, respectively. The jerk parameter appears in the fourth term of a Taylor expansion of the scale factor around $a_0$,

$$\frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{6}j_0H_0^3(t-t_0)^3 + O\left[(t-t_0)^4\right], \tag{25}$$

where the subscript 0 shows the present value. One can rewrite Eq. (23) as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H}. \tag{26}$$

Equations (22) and (26) reduce to

$$j(t) = \frac{t^3 + 3(t^2 - t + 1)}{(1+t)^3}. \tag{27}$$

This value overlaps with the value $j \simeq 2.16$ obtained from the combination of three kinematical data sets: the gold sample of type Ia supernovae,[24] the SNIa data from the SNLS project,[25] and the x-ray galaxy cluster distance measurements[26] at $t \simeq 0.05$.

Secondly, we consider the interaction between dark and barotropic fluids. For this purpose we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = Q, \tag{28}$$

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}(\rho_D + p_D) = -Q. \tag{29}$$

The quantity $Q$ expresses the interaction between the dark components. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q > 0$. $Q > 0$ ensures that the second law of thermodynamics stands fulfilled.[27] Here we emphasize that the continuity Eqs. (28) and (29) imply that the interaction term ($Q$) should be proportional to a quantity with units of inverse of time. i.e. $Q \propto \frac{1}{t}$. Therefore, a first and natural candidate can be the Hubble factor $H$ multiplied with the energy density. Following Amendola *et al.*[28] and Gou *et al.*,[29] we consider

$$Q = 3H\sigma\rho_m, \tag{30}$$

where $\sigma$ is a coupling constant. Using Eq. (30) in Eq. (28) and after integrating the resulting equation, we obtain

$$\rho_m = \rho_0 a^{-3(1+\omega_m-\sigma)}. \tag{31}$$

By using Eq. (31) in Eqs. (3) and (4), we again obtain $\rho_D$ and $p_D$ in terms of scale factor $a(t)$,

$$\rho_D = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0 a^{-3(1+\omega_m-\sigma)}, \tag{32}$$

$$p_D = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0(\omega_m - \sigma)a^{-3(1+\omega_m-\sigma)}, \tag{33}$$

respectively. Putting the value of $a(t)$ from Eq. (13) in Eqs. (32) and (33), we obtain

$$\rho_D = 3\left[\frac{(1+t)^2}{4t^2} + \frac{k}{te^t}\right] - \rho_0(te^t)^{-\frac{3}{2}(1+\omega_m-\sigma)}, \tag{34}$$

$$p_D = -\left[\frac{3(1+t)^2 - 4}{4t^2} + \frac{k}{te^t} + \rho_0\omega_m(te^t)^{-\frac{3}{2}(1+\omega_m-\sigma)}\right], \tag{35}$$

respectively. Using Eqs. (34) and (35) in Eq. (7), we can find the EoS parameter of dark field as

$$\omega_D = -\left[\frac{\frac{3(1+t)^2-4}{4t^2} + \frac{k}{te^t} + \rho_0\omega_m(te^t)^{-\frac{3}{2}(1+\omega_m-\sigma)}}{3\left[\frac{(1+t)^2}{4t^2} + \frac{k}{te^t}\right] - \rho_0(te^t)^{-\frac{3}{2}(1+\omega_m-\sigma)}}\right]. \tag{36}$$

The behavior of the EoS in term of cosmic time $t$ is shown in Fig. 4. It is observed that like the minimal coupling case, the EoS parameter is an increasing

function of time for all closed, open and flat universes, the rapidity of its increase at the early stage depends on the type of universe. At the later stage of evolution it tends to the same constant value independent of the types of the Universe. The EoS parameter of DE begins in phantom region and tends to $-1$ (cosmological constant.

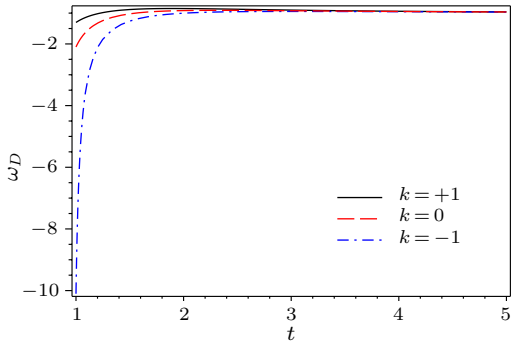



**Fig. 4.** The plot of EoS parameter vs $t$ for $\rho_0 = 10$, $\omega_m = 0.5$ and $\sigma = 0.3$.

The expressions for the matter-energy density $\Omega_m$ and dark-energy density $\Omega_D$ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{4t^2}{3(1+t)^2}\rho_0(te^t)^{-3n(1+\omega_m-\sigma)}, \quad (37)$$

and

$$\Omega_D = \frac{\rho_D}{3H^2} = 1 + \frac{4kt}{3(1+t)^2e^t} - \frac{4t^2}{3(1+t)^2} \times \rho_0(te^t)^{-3n(1+\omega_m-\sigma)}, \quad (38)$$

respectively. From Eqs. (37) and (38), we obtain

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{4kt}{3(1+t)^2e^t}, \quad (39)$$

which is the same as Eq. (19). Therefore, we observe that in interacting case the density parameter has the same properties as in the non-interacting case. The expressions for deceleration parameter and jerk parameter are also the same as in the non-interacting case.

Studying the interaction between the dark energy and ordinary matter will open up the possibility of detecting the dark energy. It should be pointed out that evidence was recently provided by the Abell Cluster A586 in support of the interaction between dark energy and dark matter.[30,31] We observe that in the non-interacting case only open and flat universes can cross the phantom region whereas in interacting case all open, flat and closed universes can cross phantom region.

In summary, we have studied the two-fluid system within the scope of a spatially homogeneous and isotropic FRW model. The role of a two-fluid system minimally or directly coupled in the evolution of the dark energy parameter has been investigated. In doing so, the scale factor is taken to be an exponential law function of time. It is concluded that in the non-interacting case only open and flat universes cross the phantom region whereas in the interacting case Sall three universes can cross the phantom region.

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