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Controllable Optical Switch in a One-Dimensional Resonator Waveguide Coupled to a Whispering-Gallery Resonator

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Single photon transport properties in a one-dimensional array of coupled microcavities waveguide coupled to a whispering-gallery resonator interacting with a Λ-type system are theoretically investigated. The calculations reveal that the transport properties of single photons with arbitrary energy can be controlled by varying the Rabi frequency and detuning the control optical field. This phenomenon can be used for controllable optical switching.

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Single photon transport properties in a one-dimensional waveguide coupled to a two-level[1−10] or multi-level[11−17] system have been studied theoretically and experimentally for their potential applications in quantum information and all-optical devices. A coupled cavity array is considered as a one-dimensional waveguide and the single photon transport properties in such a system coupled to a two-level and multi-level system have been studied. Many quantum effects, such as slowing light,[18] quantum nodes,[19] a tunable "mirrors" for single photon transport,[20,21] and single photon scattering by a controllable two-level system,[3] have been reported in such systems. However, in most studies, only the single photon with some special energy can be reflected or transmitted. This may limit its applications.

Microcavities play important roles in optical devices, quantum information and quantum computating. Several kinds of microcavites have been proposed and fabricated, such as microspheres,[22] photonic crystal nanocavities,[23] microdisk microcavities[24] and plasmonic cavities.[25] Whispering-gallery resonators (WGRs) have an ultrahigh quality factor and a small mode volume.[26,27] Based on such properties, many excellent works using WGRs have been reported. WGRs can be used for fundamental studies including cavity optomechanics,[28] cavity quantum electrodynamics,[29−31] optical switching,[32] single photon transistors,[33] second-Harmonic Generation,[34] and so on.

In this Letter, coherent controlling single photon transport properties in a one-dimensional resonator waveguide coupled to a WGR containing a Λ-type system by using the quantum-mechanical method is theoretically investigated. It is shown that one can realize single photon switching by adjusting the optical field in such a system. Our results may have applications in all-optical devices and quantum information processing.

The system of interest is exhibited in Fig. 1(a). A WGR interacting with a Λ-type system is coupled to an array-coupled microcavity waveguide. The WGR supports two degenerate modes, which are represented by a^+ (counterclockwise) and b^+ (clockwise), respectively. Figure 1(b) exhibits the energy-level configuration of the Λ-type system. A classic control optical field with frequency ω_c matches the transition $|2\rangle$ and $|3\rangle$. A photon, with modes a^+ or b^+, matches the transition $|1\rangle$ and $|2\rangle$.

The total Hamiltonian of the system is given as

$$H = H_{\rm W} + H_{\rm A} + H_{\rm I}, \tag{1}$$

where $H_{\rm W}$ and $H_{\rm A}$ describe the propagation of the photon and the free Λ-type three-level atom, respectively; $H_{\rm I}$ denotes the controlling couplings term. With the rotation wave approximation, one can obtain[3,9,15]

$$\begin{aligned} H_{\rm W} =& \omega_c \sum\nolimits_j C_j^+ C_j - \xi_0 (C_j^+ C_{j+1} + C_{j+1}^+ C_j) \\ & - \xi_1 \sum\nolimits_j (C_0^+ a + a^+ C_0 + C_0^+ b + b^+ C_0) \\ & + \omega_{ab} (a^+ a + b^+ b), \end{aligned} \tag{2}$$

$$H_{\rm A} = \omega_2 |2\rangle\langle 2| + (\omega_2 - \Delta)|3\rangle\langle 3|, \tag{3}$$

$$\begin{aligned} H_{\rm I} =& h(a^+ b + b^+ a) + g_a (a^+ |1\rangle\langle 2| + |2\rangle\langle 1| a) \\ & + g_b (b^+ |1\rangle\langle 2| + |2\rangle\langle 1| b) + \frac{\Omega}{2} (|2\rangle\langle 3| + |3\rangle\langle 2|). \end{aligned} \tag{4}$$

Here ω_c and ω_{ab} are the circular frequencies of the microcavities of the 1D array waveguide and of the degeneration modes of a^+, b^+ of WGR, respectively; ω_2 and ω_3 are the circular frequency of energy level $|2\rangle$, $|3\rangle$, respectively. We assume that all the cavities of the array waveguide have the same frequency and set the energy scale such that the energy of level $|1\rangle$ is zero. $C_j^+(C_j)$ is the bosonic creation (annihilation) operator of the jth cavity mode; ξ_0 is the hopping energy between two nearest-neighbor cavities; ξ_1 is the coupling strength between the zeroth cavity and the WGR; h is the intermode backscattering strength; $g_{a/b}$ is the coupling strength between the Λ-type system

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and mode a/b of the cavity; Ω is the Rabi frequency of the control field, and Δ is the detuning.

The stationary eigenstate for the Hamiltonian can be expressed as[3,15]

$$|E\rangle = \sum_j u_k(j)C_j^+|0,1\rangle + u_a a^+|0,1\rangle + u_b b^+|0,1\rangle + u_2|0,2\rangle + u_3|0,3\rangle, \quad (5)$$

where $|0,1\rangle$ represents the state in which there is an atom in the ground state and no photons in the array cavities and the WGR; $|0,2\rangle$ and $|0,3\rangle$ denote the atom in the state $|2\rangle$ and $|3\rangle$ while no photon in the cavities and the WGR; $u_k(j)$, u_a, u_b, u_2, u_3 are the probability amplitudes of each state accordingly.

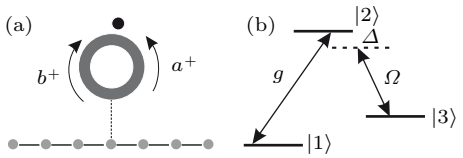


Fig. 1. (Color online) (a) Sketch of an optical system consisting of an array waveguide coupled to a whispering-gallery resonator containing a Λ-type system. The black dot denotes the atom. The gray dots and the ring represent the microcavities of the waveguide and the WGR, respectively. Here a^+ and b^+ are the two degenerate whispering-gallery modes. (b) The energy configuration of the Λ-type system, where $|1\rangle$ and $|2\rangle$ are coupled by the resonator; $|3\rangle$ and $|2\rangle$ are driven by a classic optical field.

From the eigen-equation $H|E\rangle = E|E\rangle$, one can obtain the equations of motion as follows:

$$(\omega_c - E)u_k(j) = \xi_1(u_a + u_b)\delta_{j,0} + \xi_0(u_k(j+1) + u_k(j-1)), \quad (6)$$

$$(\omega_{ab} - E)u_a = \xi_1 u_k(0) - h u_b - g_a u_2, \quad (7)$$

$$(\omega_{ab} - E)u_b = \xi_1 u_k(0) - h u_a - g_b u_2, \quad (8)$$

$$(\omega_2 - E)u_2 = -g_a u_a - g_b u_b - \frac{\Omega}{2}u_3, \quad (9)$$

$$(\omega_2 - \Delta - E)u_3 = -\frac{\Omega}{2}u_2. \quad (10)$$

The energy E of the incident photon obeys the dispersion relation

$$E = \omega_c - 2\xi_0 \cos k. \quad (11)$$

By eliminating amplitudes for the states $|2\rangle$ and $|3\rangle$, we obtain the discrete scattering equation for the amplitude of the single photon,

$$(\omega_c - E - V(j))u_k(j) = \xi_0(u_k(j+1) + u_k(j-1)), \quad (12)$$

where the effective potential is given by

$$V(j) = \frac{2\xi_1^2(4BC - \Omega^2)}{4ABC - A\Omega^2 - 8g^2C}\delta_{j,0} = V\delta_{j,0}. \quad (13)$$

We have assumed the coupling strength $g_a = g_b = g$. A, B and C in Eq. (13) are defined as $A \equiv E - \omega_{ab} - h$, $B \equiv E - \omega_2$ and $C \equiv E - \omega_2 + \Delta$. It is obvious that the effective potential is determined by the inter structure of the Λ-type system and the classic control field.

The usual solution to the scattering equation for $j \neq 0$ can be written as

$$u(j) = \begin{cases} e^{-ikj} + re^{ikj}, & j < 0, \\ te^{-ikj}, & j > 0, \end{cases} \quad (14)$$

where r and t are the reflection and transmission amplitudes, respectively.

By using the continuous condition $u(0^+) = u(0^-)$ and the scattering equation at the zeroth site

$$(\omega_c - E - V(0))u_k(0) = \xi_0(u_k(1) + u_k(-1)), \quad (15)$$

one can obtain the reflection and transmission amplitudes, which are given as

$$r = \frac{V}{2i\xi_0 \sin k - V}, \quad (16)$$

$$t = \frac{2i\xi_0 \sin k}{2i\xi_0 \sin k - V}. \quad (17)$$

Without the control optical field, the Λ-type system degenerates to a two-level system. The single photon transport properties in the coupled system with the two-level system have been stated by Zang and Jiang recently.[9] Here we can find that by using a Λ-type three-level system, one can control the transport properties of single photons with arbitrary energy, which may have wider applications.

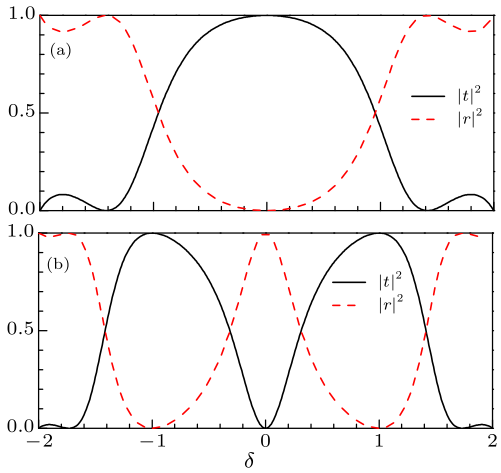


Fig. 2. Single photon transmission and reflection spectrums as a function of $\delta \equiv 2\xi_0 \cos k$ with controlling optical field turning off (a) and on (b). In the calculation, $h = 0$, $\Delta = 0$, $g = \xi_1 = \xi_0$, $\omega = \omega_{ab} = \omega_2 = 3\xi_0$. Ω is taken as $\Omega = 2\xi_0$ in Fig. 2(b).

We firstly investigate controlling the single photon transport without dissipation. As shown in Eqs. (16) and (17), the scattering properties of a single photon with definite momentum is determined by the potential $V(j)$, which can be tuned by Ω and Δ. When $V(j) = 0$, the reflection amplitude $r = 0$, which corresponds to perfectly transmission. When $V(j)$ takes the

singularities (i.e. denominator of $V(j)$ is equal to 0), the reflection amplitude $r = -1$, which corresponds to perfect reflection. From Eq. (16), one can obtain that the conditions of perfect transmission read

$$E = \omega_2 \pm \frac{\sqrt{\Omega^2 + \Delta^2}}{2}. \tag{18}$$

Equation (18) shows that the positions of transmission peak is dependent on ω_2, Ω and Δ. Tuning the Ω and Δ, one can realize perfect transmission.

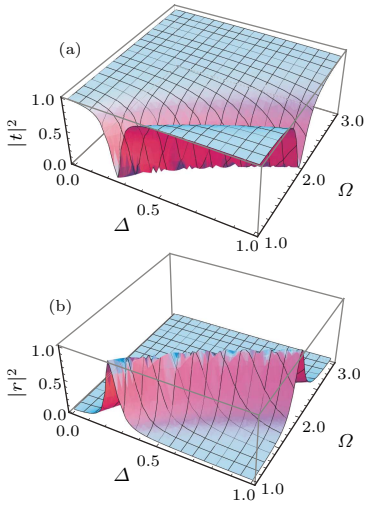


Fig. 3. (Color online) Three-dimensional plot of transmission (a) and reflection (b) probabilities vs Rabi frequency Ω and detuning Δ. In the calculation, $h = \xi_0, \omega = 4\xi_0$, $\omega_{ab} = 3\xi_0$, $\omega_2 = 2\xi_0$ and $2\xi_1 \cos k = \xi_0$. Ω and Δ are in the units of ξ_0.

When $\omega_{ab} = \omega_2$, $\Delta = h = 0$, one can obtain perfect reflection when

$$E = \omega_2 + \frac{\sqrt{\Omega^2 + 8g^2}}{2},\ \omega_2 - \frac{\sqrt{\Omega^2 + 8g^2}}{2},\ \omega_2. \tag{19}$$

The reflection spectrum shows three peaks. The position of the reflection peaks can also be tuned by varying the Ω of the classic control field. We note that when $\Omega = 0$, the Λ-type three-level system degenerates into a two-level system. The single photon with energy $E = \omega_2$ is perfectly transmitted.[9] When $E = \omega_2$, the incident single photon, the WGR and the atom are on resonant. The on-resonance photon completely transmits through the system, which is analogous to electromagnetically induced transparency (EIT) phenomena.[9] The EIT medium placed in a cavity can exhibit many quantum effects such as slow light,[35] cavity linewidth narrowing,[36,37] and so on. When $\Omega \neq 0$, the single photon with energy $E = \omega_2$ is perfectly reflected. Thus one can control the single photon with $E = \omega_2$ being reflected or transmitted by tuning on or tuning off the control field, as indicated in Fig. 2. These properties can be used to realize single photon switching.

For a more general case, $\omega_{ab} \neq \omega_2$ and $h \neq 0$, we can also control single photon transport properties by tuning Ω and Δ. If the conditions

$$\begin{aligned} &(E - \omega_2)(E - \omega_2 + \Delta) = \Omega^2 \\ &(E - \omega_2)(E - \omega_2 + \Delta) \geq 0 \end{aligned} \tag{20}$$

are satisfied, a photon with arbitrary energy is perfectly transmitted.

For the incident photon with $E \neq \omega_{ab} + h$, one can fine tune Ω and Δ to satisfy the following conditions to obtain the perfect reflection:

$$\begin{aligned} &\big[4(E - \omega_{ab} - h)(E - \omega_2)(E - \omega_2 + \Delta) \\ &- 8g^2(E - \omega_2 + \Delta)\big](E - \omega_{ab} - h)^{-1} = \Omega^2, \\ &\big[4(E - \omega_{ab} - h)(E - \omega_2)(E - \omega_2 + \Delta) \\ &- 8g^2(E - \omega_2 + \Delta)\big](E - \omega_{ab} - h)^{-1} \geq 0. \end{aligned} \tag{21}$$

If the incident photon energy $E = \omega_{ab} + h$, the perfect reflection conditions become $\Delta = \omega_2 - E$, which is independent of Rabi frequency Ω.

In Figs. 3(a) and 3(b), we plot the reflection and transmission probabilities, respectively. The figures clearly show that one can fine tune one of the parameters Ω or Δ to reach perfect reflection or transmission when the other parameter is fixed.

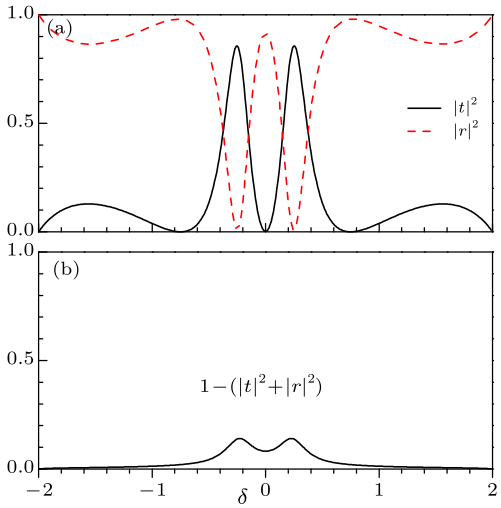


Fig. 4. (a) Single photon transmission and reflection spectrum as a function of $\delta \equiv 2\cos k$ with dissipation. (b) The energy loss ratio as a function of δ. In the calculation, $h = 0$, $\Delta = 0$, $g = \Omega = \xi_0$, $\omega = \omega_{ab} = \omega_2 = \xi_1 = 2\xi_0$; $\gamma_c = \gamma_2 = \gamma_3 = 0.04\xi_0$.

It is difficult to avoid the dissipative processes in the realistic physical system. We replace A, B, and C by $A_d \equiv A + i\gamma_c$, $B_d \equiv B + i\gamma_2$, and $C_d \equiv C + i\gamma_3$[19] to obtain the reflection amplitude and transmission amplitudes, which are given as

$$r = \frac{V_d}{2i\xi_0 \sin k - V_d}, \tag{22}$$

$$t = \frac{2i\xi_0 \sin k}{2i\xi_0 \sin k - V_d}, \tag{23}$$

where $V_d = 2\xi_1^2(4B_dC_d - \Omega^2)/(4A_dB_dC_d - A_d\Omega^2 - 8\,g^2C_d)$. γ_c, γ_2, and γ_3 are the decays rates of the WGR, states $|2\rangle$ and $|3\rangle$, respectively.

Figure 4 shows the reflection and transmission probabilities with decay. It is obvious that perfect reflection can not be obtained even if $|t|^2 \approx 0$. The maximum value of transmission probability is less than 1, as shown in Fig. 4(a). The decoherence effects, which result from the inelastic scattering process, have been studied with the harmonics oscillator model recently.[38] Generally speaking, dissipation lowers the peaks of the reflection and transmission spectra. However, one can still obtain perfect reflection and transmission under some special conditions, although dissipation still exists.[38,39] In the Λ-type three-level system, by tuning the Rabi frequency Ω and Δ, we can control the positions of the transmission and reflection peak. Figures 5(a) and 5(b) exhibit the reflection and transmission probabilities varying with Ω and Δ with atomic and cavity decay. It is clearly shown that for a photon incident with fixed energy, one can finely tune the Ω and Δ to control the photon transport.

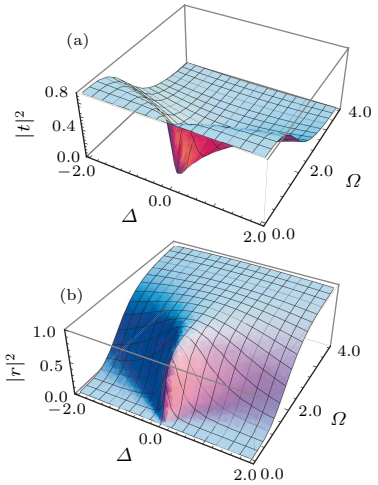


Fig. 5. (Color online) Three-dimensional plot of transmission (a) and reflection (b) probabilities vs Rabi frequency Ω and detuning Δ. In the calculation, $h = 0$, $\gamma_c = \gamma_2 = \gamma_3 = 0.04\xi_0$, $g = \xi_0$, $\omega = \omega_{ab} = \omega_2 = 2\xi_0$, $\cos k = 0$.

In summary, we have theoretically investigated coherent controlling single photon transport in a 1D cavity-array waveguide coupled to a whispering-gallery resonator interacting with a Λ-type system. A single photon with arbitrary energy can be perfectly reflected or transmitted by tuning the Rabi frequency and detuning the control optical field. By tuning the Rabi frequency and detuning the control optical field, one can still control the positions of reflection and transmission peaks for a photon with arbitrary energy. Our results may have applications in optical switching and quantum information processing.

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