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Spiral Wave Generation in a Vortex Electric Field *

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The effect of a vortical electric field on nonlinear patterns in excitable media is studied. When an appropriate vortex electric field is applied, the system exhibits pattern transition from chemical turbulence to spiral waves, which possess the same chirality as the vortex electric field. The underlying mechanism of this is discussed. We also show the meandering behavior of a spiral under the taming of a vortex electric field. The results obtained here may contribute to control strategies of patterns on surface reaction.

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Spiral waves are one of the most common and widely studied patterns in nature. They appear in hydrodynamic systems, chemical reactions and a large variety of biological, chemical and physical systems.[1−5] Much attention has been paid to their rich nonlinear dynamics, as well as potential applications in various biological or physiological systems, since the emergence and instability of spirals usually lead to abnormal states, for example in cardiac arrythmia[6,7] and epilepsy.[8] Much research has been carried out in studying pattern formations in catalytic CO oxidation on Pt(110),[9−11] because they provide practical utilization in industry. A rich variety of spatiotemporal patterns, including travelling pulses, standing waves, target patterns, spiral waves and chemical turbulence have been observed in this system.[12−16]



Fig. 1. The evolution of spiral waves induced by clockwise-rotating VEF with $\Gamma = 2.6$, $\varepsilon = 0.05$, and $R = 1.2$: (a) $t = 0$; (b) $t = 200$; (c) $t = 400$; (d) $t = 1000$.

Under the forcing of external field, the pattern tends to organize itself to the pattern with the same symmetry of the applied field.[17−20] For example, in time and space symmetry[21,22] the spiral[23] and Turing strips[24] evolve into hexagonal patterns that are closer to the rotation symmetry of the imposed circularly polarized electric field. In this Letter, we investigate the evolution and transition of spatiotemporal chaos to spirals resulting from the vortex flow, by imposing a vortical electric field (VEF). The meandering of spiral waves under the influence of VEF will also be studied. The underlying mechanism regarding the transition of patterns will also be discussed. It will be shown that the VEF is a prominent method that can be used to tame patterns in surface reactions.

We consider the effect of the vortical electric field on an excitable media, which is described by a modified FitzHugh–Nagumo model (the Bär model).[12] The two variable reaction-diffusion model, with additional electric term $\boldsymbol{E}\cdot\nabla u$, is given by

$$\begin{aligned}\frac{\partial u}{\partial t} &= f(u,v) + \nabla^2 u + \boldsymbol{E}\cdot\nabla u,\\ \frac{\partial v}{\partial t} &= g(u,v).\end{aligned} \tag{1}$$

The two-dimensional Laplace operator $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and the variables u and v can be viewed as the "fast" and "slow" variables. In this model, $f(u,v) = \frac{1}{\varepsilon}u(1-u)[u-(v+b)/a]$, and $g(u,v)$ describe a delayed production of inhibitors with $g(u,v) = -v$ for $0 \le u < 1/3$, $g(u,v) = 1 - 6.75u(u-1)^2 - v$ for $1/3 \le u < 1$, and $g(u,v) = 1 - v$, for $u > 1$. ε is the ratio of their temporal scales, which characterizes the excitability of the medium. The parameters $a = 0.84$ and $b = 0.07$ are fixed.

A vortex electric field could be realized in experiments.[19] For example, a long and straight

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electric solenoid will generate a VEF by inducing a changeable magnetic field. Therefore, we could discuss using the vortex electric current, which induces, for example, transition between spiral waves and other states, the homogeneous sates or turbulence, etc. Its intensity E is described by

$$\oint_L \boldsymbol{E} \cdot d\boldsymbol{l} = -\iint_s \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{s}, \tag{2}$$

where B is the intensity of the magnetic field, and the function $d\boldsymbol{B}/dt$ describes the variation of the external magnetic field. Following Eq. (2), we obtain the following expression when we consider that the area of the magnetic field is small,

$$E = -\frac{R^2}{2r}\frac{dB}{dt} = -\frac{KR^2}{2r}\frac{dI}{dt}, \tag{3}$$

where R is the radius of the solenoid. I is external electric current that is input to the solenoid; K is a proportional coefficient describing B and is in direct proportion to I; $r = \sqrt{(x-x_0)^2 + (y-y_0)^2}$, x_0 and y_0 represent the coordination origin, and (x, y) defines the site position of the field. In this case, $r > R$, i.e. the studied point is outside the solenoid, and an electric field $E = -KR^2C/2r = -\Gamma/r$ (the electric current $dI/dt = C$) is investigated. Clearly, by taking into account the effect of a counterclockwise VEF, the vortex electric field can be divided into two directions along the x- and y-axes: $E_x = E\sin\theta = -\Gamma y/r^2$, $E_y = -E\cos\theta = \Gamma x/r^2$. Here θ is the angle of the field point with respect to the positive x-axes. Thus the polarized medium is described by

$$\begin{aligned} \frac{\partial u}{\partial t} &= f(u,v) + \nabla^2 u + E_x\frac{\partial u}{\partial x} + E_y\frac{\partial u}{\partial y}, \\ \frac{\partial v}{\partial t} &= g(u,v). \end{aligned} \tag{4}$$

Inside the solenoid, we set $u = v = 0$. For simulations in 2D, zero-flux conditions have been employed in both the medium and solenoid interface boundaries. A spatial discretization of $\Delta x = \Delta y = 0.3906$ on a 256×256 array with fixed time step $\Delta t = 0.02$ has been used in an explicit Euler scheme.

Now we study the influences of the VEF on the dynamical behaviors of nonlinear patterns. In Fig. 1(a) we select a chaotic state as an initial state. Then the VEF (with an intensity of $\Gamma = 2.6$), induced by a solenoid located on the center of the medium, is imposed on. With the evolution of time, from Fig. 1(b), one can see that a small spiral is generated in the middle of the disk at about $t = 200$. Gradually, the small spiral wipes away from the turbulence and simultaneously grows into the outer region, which can be seen in Fig. 1(c). Eventually, the medium is occupied by a well developed single spiral (see Fig. 1(d)). Therefore, the VEF can develop spirals to suppress the surrounding turbulence, which may act as an efficient control method to terminate the chaotic state.

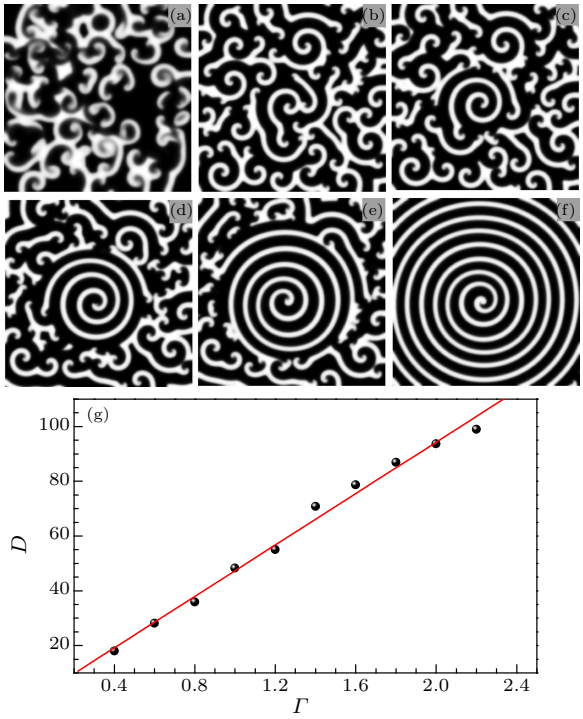


Fig. 2. (a)–(f) Patterns induced by different intensities of VEF with $\varepsilon = 0.05$ and $R = 1.2$ at $t = 500$. (a) $\Gamma = 0$; (b) $\Gamma = 0.6$; (c) $\Gamma = 0.8$; (d) $\Gamma = 1.2$; (e) $\Gamma = 1.6$; (f) $\Gamma = 2.8$. (g) The diameter of the developed spiral as a function of the amplitude of the VEF.

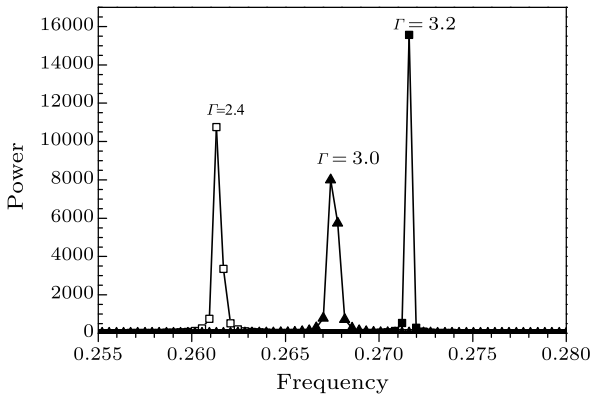


Fig. 3. The power spectra obtained from the FFT method at different Γ VEF amplitudes. The other parameters are the same as those in Fig. 1.

In the VEF there is only one parameter, that is the intensity, which can be tuned. From this point, the control method is very simple. In simulation it is found that there is a critical value of VEF intensity, below which the spiral can not be generated. In Fig. 2(a)–(f), we present the states after long-time evolution at $t = 500$. It is evident that the stronger the imposed VEF, the larger the size of the developed spiral. In Fig. 2(g), we show the dependence of the diameter of the developed spiral on the amplitude of the VEF.

Then, one can use the VEF to suppress turbulence and develop a spiral with the required sizes.

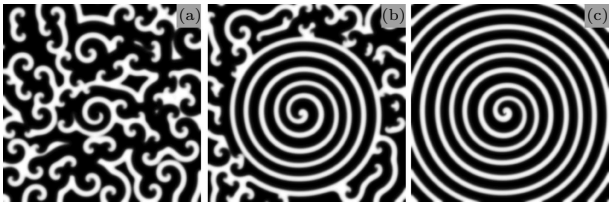


Fig. 4. Pattern evolution induced by a counterclockwise-rotating VEF with $\Gamma = 2.8$, $\varepsilon = 0.05$: (a) $t = 0$; (b) $t = 300$; (c) $t = 1000$.

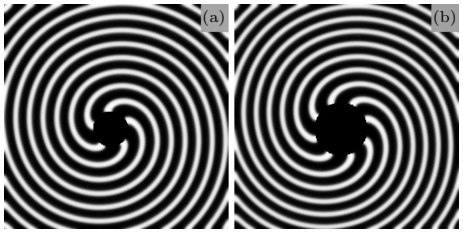


Fig. 5. A multi-arm spiral developed by the vortex field with a large radius. $\Gamma = 5.0$, $\varepsilon = 0.05$. (a) $R = 7.8$; (b) $R = 11.7$.

To give some insight into the mechanism of the suppression of turbulence by a developing spiral wave, we present the power spectra obtained from the FFT method at different Γ VEF amplitudes in Fig. 3. The data is plotted at the same spatial point. Without VEF, the turbulence has a background frequency of $f_0 = 0.238$. After a VEF is applied on the medium, a spiral is developed with a frequency that depends on the amplitude of the VEF. If the VEF is strong enough to generate a spiral with a frequency higher than f_0, the induced spiral will wipe away from the surrounding turbulence. Figure 3 verifies this point. One can find that the spiral frequency is higher than f_0 and increases with Γ amplitude, from $f = 0.261$ ($\Gamma = 2.4$), $f = 0.268$ ($\Gamma = 3.0$), to $f = 0.271$ ($\Gamma = 3.2$). If the VEF is so weak that its corresponding spiral frequency is smaller than f_0, no spiral will grow. This is the reason why there is a critical value of Γ to suppress turbulence.

The amplitude of the VEF is decreased along the radius to the outside. Due to the fact that the frequency of the spiral shows VEF amplitude dependence, its value decreases accordingly. Thus, the frequency of the spiral is inhomogeneous, that is, from a higher value to f_0. When the frequency of the spiral in the edge region is decreased to f_0, the spiral can no longer grow again, and a state described by spiral-turbulence coexistence is observed. Thus it is clear that the stronger VEF leads to higher frequency and larger sizes of developed spiral, which can be seen in Fig. 2.

It is known that the VEF has its chirality: a clockwise or counterclockwise vortex field. From the simulation, it is always shown that the process is the growth of a single spiral wave. Thus, we pay attention to the initial state when the VEF is imposed. Many small spirals form the background chaotic state. Generally, one small spiral has two ends with opposite chirality. One pattern tends to organize itself to the pattern with the same symmetry of the applied field. Once the VEF is switched on, one end of the spiral has the same chirality as the developed VEF. In Fig. 4, we show the result under the control of the counterclockwise VEF. The counterclockwise rotating spiral verifies our point.

If the radius of the solenoid is increased, compared to the small radius cases, there are many small spirals available that can be taken as seeds. Consequently, based on the mechanism discussed above, multi-arm spirals will be developed, and this is confirmed in Fig. 5. It is observed that the larger the radius, the greater the number of spiral arms. For example, a five-arm spiral is shown in Fig. 5(a) when R is 7.8 (20 grids), while an eight-arm spiral is observed in Fig. 5(b) as R is up to 11.7 (30 grids).

As a less-investigated external field, it is interesting to study the tip motion under its influences.[25,26] In Fig. 6, we present two types of motion. We select an outward meandering spiral as the initial state. The spiral rotates clockwise with two frequencies that can be indicated by the compound motion of the spiral tip. In Fig. 6(a), the clockwise trajectory (see the arrow a) showed by r_1 and r_2 is the initial state without control. The counterclockwise VEF will introduce the third frequency, which gives another counterclockwise motion (see the arrow b) indicated by the trajectory with radius r_3. With stronger amplitude, VEF reverses the second motion by increasing the second frequency, which leads to the changes from outward meandering to inward meandering, and clockwise to counterclockwise rotation. This result is shown in Fig. 6(b).

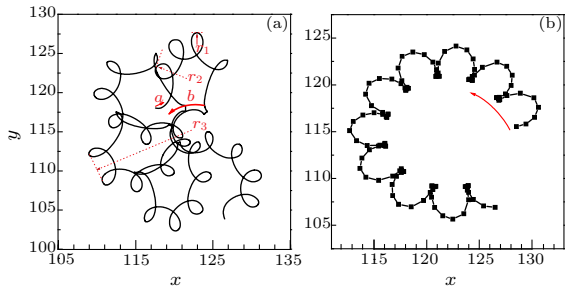


Fig. 6. (a) The trajectories of the motion of the spiral tip. The clockwise trajectory (see the arrow a) showed by r_1 and r_2 is the initial state without control. Another, counterclockwise, motion (see the arrow b) indicated by the trajectory with radius r_3 is induced by the counterclockwise VEF. (b) The inward-meandering trajectory of the second motion modulated by the VEF.

In conclusion, we suggested a control method to suppress spatial-temporal chaos in excitable media. The size and frequency of the developed spirals are

found to increase with the intensity of the imposed VEF. When the radius of the solenoid is increased, multi-arm spirals can be generated. It is found that the chirality of the generated spiral is always the same as the imposed VEF. The VEF can also be utilized to tune the dynamics of meandering spirals. One can easily develop spiral waves of the required size by modulating the intensity of the VEF. It is expected that our results will contribute to the understanding of chemical reaction-diffusion systems, especially in CO oxidation on a Pt surface.

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