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Area Spectra of Schwarzschild-Anti de Sitter Black Holes from Highly Real Quasinormal Modes *

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Motivated by the new physical interpretation of quasinormal modes proposed by Maggiore [Phys. Rev. Lett. 100 (2008) 141301], we investigate the quantization of large Schwarzschild-Anti de Sitter black holes in even-dimensional spacetimes, from the interesting highly real quasinormal modes found recently. Following Maggiore's treatment and Kunstatter's method, we derive the area and entropy spectra of the black holes. It is found that the results from both approaches are in full consistency. This implies that one can quantize a black hole via different asymptotic quasinormal modes besides the high damping ones that are usually adopted in the literature. Furthermore, we find that the area and entropy spectra are equidistant and independent of the cosmological constant. However, the spacings depend on the black hole dimension.

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Since Bekenstein[1] firstly conjectured the equidistant area spectrum $A_n = \gamma n\hbar$ $(n = 1, 2, 3, \ldots)$ by regarding the horizon area of a nonextremal black hole as a classical adiabatic invariant, many attempts have been made to derive the area and entropy spectra directly from the dynamical modes of the classical theory.[2–8] An important step in this direction was made by Hod,[9] who suggested that the real part of the asymptotic quasinormal modes (QNMs) be regarded as a transition frequency in the semiclassical limit and then determined the dimensionless constant γ as $\gamma = 4\ln 3$ from QNMs of the Schwarzschild black hole. Later, Kunstatter[10] obtained an equally spaced entropy spectrum of the $d(\geq 4)$-dimensional Schwarzschild black hole, which confirmed Hod's idea. In this work, the quantity $I = \int dM/\omega_R$ is treated as the adiabatic invariant, which has an equidistant spectrum in the semiclassical limit according to the Bohr–Sommerfeld quantization $I = n\hbar$. These developments together with the possible links of QNMs to loop quantum gravity, as proposed by Dreyer,[11] spurred lots of subsequent research.

However, Hod's calculation is not universal since it depends on the asymptotic behavior of the black hole spacetime under study, as pointed out by Natario.[12] To avoid some difficulties raised by Hod's idea, Maggiore[13] put forward a new physical interpretation for the QNMs of black holes.

According to Maggiore's proposal, black hole perturbations should be considered in terms of a collection of damped harmonic oscillators and the more relevant physical frequency connecting QNMs to the quantum structure of the horizon area is the proper frequency ω_c of the umdamped oscillators rather than the frequency of the damped oscillation, which agrees with the real part of the QNM frequency. Since the proper frequency ω_c is of the form $\omega_c = \sqrt{\omega_R^2 + \omega_I^2}$, we have $\omega_c \approx |\omega_I|$ rather than $\omega_c \approx |\omega_R|$, in the high damping limit. Maggiore's interpretation leads to results that confirm Bekenstein's original conjecture but differ from Hod's. This rejuvenates a great interest in quantizing the black hole area via the new interpretation from the asymptotic highly damped QNMs.[14–22] Taking the transition frequency as $\omega = (\omega_I)_n - (\omega_I)_{n-1}$, Vagenas[14] observed an equally spaced area spectrum of Kerr black hole using Maggiore's treatment. However, the spectrum is not equidistant when the method of Kunstatter is employed, because there is a logarithmic correction term in the formula of adiabatic invariant. Meanwhile, Medved[15] pointed out that by taking the limit $J \ll M^2$ into account, both methods would give the same spectrum, which is independent of the angular momentum. Moreover, he argued further that the quantum number n in the Bohr–Sommerfeld quantization condition is actually a measurement of the areal deviation from extremity, rather than the area itself. This implies that Kunstatter's method should be restricted to cases far from the extremal black holes. Furthermore, Wei *et al.*[16] investigated the area/entropy spectra of various types of non-rotating black holes with no charge in different dimensions. They claimed that the entropy spectrum is equidistant and independent of the spacetime dimension, while the spacing of the area spectrum depends on gravity theory. In Einstein's gravity, it is equally spaced, otherwise non-equidistant. However, it is worth noting that the area and entropy spectra for 3-dimensional non-rotating BTZ black hole are

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relevant to the cosmological constant. More recently, Kwon[17] obtained equally spaced area and entropy spectra, irrelevant to the cosmological constant, for a 3-dimensional non-rotating BTZ black hole, when employing the action variable quantization rather than the adiabatic invariant quantization.

In addition, black holes in Anti-de Sitter (AdS) space have attracted a great deal of attention owing to the AdS/CFT correspondence. The QNMs of AdS black holes have been calculated extensively with various numerical or analytical methods (see Ref. [24] for a recent review). Daghigh[25,26] investigated the asymptotic QNMs of Schwarzschild-AdS (SAdS) black holes using complex coordinate techniques and found that there are interesting regions of highly real QNMs, which did not appear in the earlier calculations. Therefore, it is of interest to investigate the area and entropy spectra of SAdS black holes with these novel QNMs.

The line element of a D-dimensional SAdS black hole is given by

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_{D-2}^2, \quad (1)$$

with

$$f(r) = 1 - \frac{2\mu}{r^{D-3}} + \lambda r^2. \quad (2)$$

The ADM mass M of the black hole is related to the parameter μ via

$$M = \frac{(D-2)A_{D-2}}{8\pi G_D}\mu, \quad (3)$$

where G_D means the Newton gravitational constant in D-dimensional spacetime and $A_{D-2} = 2\pi^{(D-1)/2}\,\Gamma[(D-1)/2]$ is the area of a unit $(D-2)$-dimensional sphere. The cosmological constant Λ is given by

$$\Lambda = \frac{(D-1)(D-2)}{2}\lambda = -\frac{(D-1)(D-2)}{2l^2}, \quad (4)$$

where $l = 1/\sqrt{\lambda}$ is the AdS radius. For large black holes, the warp factor $f(r)$ reduces to

$$f(r) = -\frac{2\mu}{r^{D-3}} + \lambda r^2. \quad (5)$$

The event horizon can be determined from the roots of $f(r) = 0$, which reads

$$r_h = (2\mu/\lambda)^{1/(D-1)}. \quad (6)$$

Accordingly, the Hawking temperature is given by

$$T_H = \kappa_h/2\pi = f'(r_h)/4\pi = \frac{D-1}{4\pi}\lambda r_h, \quad (7)$$

where the prime denotes the differentiation with respect to r and κ_h is the surface gravity of the event horizon.

Recently, Daghigh[25,26] investigated the asymptotic QNMs of SAdS black holes using the complex coordinate WKB methods. The results show that for large black holes, the QNMs of scalar, vector and tensor perturbations in any dimension ($D > 3$) can be expressed as

$$\omega\eta = n\pi + \frac{\pi}{4}(D+1) - \frac{i}{2}\ln\Big(\frac{D-2}{D-3}\Big), \quad (8)$$

except scalar perturbations in D-dimensional spaces ($D = 4, 5$); η in Eq. (8) is a multivalued constant, which is related to the branch cuts chosen in the calculations and is given by

$$\eta = |\eta|e^{\frac{i\pi}{D-1}},\ |\eta|e^{\frac{3i\pi}{D-1}},\ |\eta|e^{\frac{5i\pi}{D-1}}, \ldots, |\eta|e^{\frac{(2D-3)i\pi}{D-1}}, \quad (9)$$

where

$$|\eta| = \frac{1}{4T_H \sin(\frac{\pi}{D-1})}. \quad (10)$$

In even-dimensional spacetimes, the QNMs turn out to be

$$\omega = 4T_H \sin\Big(\frac{\pi}{D-1}\Big)\Big[n\pi + \frac{\pi}{4}(D \pm 1) - \frac{i}{2}\ln\frac{D-2}{D-3}\Big]. \quad (11)$$

For the special case of dimension 4, the QNM frequency of vector and tensor perturbations can be evaluated as

$$\omega = 4T_H \sin(\frac{\pi}{3})\Big[n\pi \pm \frac{\pi}{4} - \frac{i}{2}\ln 2\Big], \quad (12)$$

which is in agreement with Eq. (11), while the QNM frequency of scalar perturbations should be of the form

$$\omega = 4T_H \sin\Big(\frac{\pi}{3}\Big)\Big[n\pi + \frac{\pi}{2} \pm \frac{\pi}{4} - \frac{i}{2}\ln 2\Big]. \quad (13)$$

It is obvious from Eqs. (11)–(13) that the real part ω_R tends to infinity, whereas the damping rate ω_I approaches a finite value, in the large overtone numbers limit. That is why it is defined as the highly real QNMs. Note that these highly real QNMs only appear in even dimensions and have not been found in previous calculations.

We will derive the area and entropy spectra of even-dimensional large SAdS black holes from the highly real QNMs via the new interpretation. Two approaches, Kunstatter's method and Maggiore's treatment, will be utilized respectively. Since the highly real QNMs only emerge in even-dimensional black holes ($D \geq 4$), we will only concentrate on the quantum structures of these cases.

Given a system with energy E and vibrational frequency $\omega(E)$, one can regard the quantity

$$I = \int \frac{dE}{\omega(E)} \quad (14)$$

as an adiabatic invariant, as proposed by Kunstatter.[10] According to the Bohr–Sommerfeld

quantization condition, an adiabatic invariant corresponds to an equidistant spectrum, i.e. $I = n\hbar$. For a harmonic oscillator, the frequency $\omega(E)$ is the vibrational frequency that corresponds to the system's energy E under a slow variation of a parameter related to the energy, then we can obtain a small variation energy dE and take the quantity E/ω as an adiabatic invariant. From Maggiore's proposal, the proper frequency should be expressed as $\omega_c = \sqrt{\omega_R^2 + \omega_I^2}$. This implies that we will have $\omega_c \approx \omega_R$ in the large n limit. In the present framework, we consider the transition $n \to (n-1)$ for the even D-dimensional large SAdS black holes. It is not difficult to write, from Eqs. (11)–(13), the transition frequency as

$$\omega = (\omega_R)_n - (\omega_R)_{n-1} = 4\pi T_H \sin\left(\frac{\pi}{D-1}\right) \tag{15}$$

for all types of perturbations in any even $D \geq 4$ dimensional spacetimes.

Adopting the mass definition $M = E$[27] and taking Eqs. (3) and (6) into account, we can write the absorbed energy as

$$\begin{aligned} dM &= \left[\frac{(D-2)A_{D-2}}{8\pi G_D}\right] d\mu \\ &= \left[\frac{(D-2)A_{D-2}}{8\pi G_D}\right] \frac{\lambda(D-1)}{2} r_h^{D-2} dr_h. \end{aligned} \tag{16}$$

According to the Bohr–Sommerfeld quantization condition, we obtain

$$I = \int \frac{dM}{\omega(E)} = \frac{r_h^{D-2} A_{D-2}}{16\pi G_D \sin[\pi/(D-1)]} = n\hbar. \tag{17}$$

Then the event horizon radius can be expressed as

$$r_h = \left\{\frac{16n\pi\hbar G_D \sin[\pi/(D-1)]}{A_{D-2}}\right\}^{1/(D-2)}. \tag{18}$$

For large SAdS black holes, the horizon area can be written as

$$A_n = A_{D-2} r_h^{D-2} = 16n\pi\hbar G_D \sin(\frac{\pi}{D-1}). \tag{19}$$

Thus the area quanta should be

$$\Delta A = A_n - A_{n-1} = 16\pi\hbar G_D \sin(\frac{\pi}{D-1}). \tag{20}$$

According to the thermodynamics of black hole in AdS space,[21] the entropy spectrum reads

$$\Delta S = \frac{\Delta A}{4\hbar} = 4\pi G_D \sin(\frac{\pi}{D-1}). \tag{21}$$

Then we obtain the area and entropy spectrum for even D-dimensional large SAdS black holes, following Kunstatter's method.

Next, we will derive the area and entropy spectrum from Maggiore's treatment to check whether the two methods match in the present case.

The horizon area of a large SAdS black hole can be written as

$$A = A_{D-2} r_h^{D-2} \approx A_{D-2}\left(\frac{2\mu}{\lambda}\right)^{(D-2)/(D-1)} \tag{22}$$

and the variation of the area is

$$\Delta A \approx \frac{2A_{D-2}}{\lambda}\frac{D-2}{D-1}\left[(\frac{2\mu}{\lambda})^{-1/(D-1)}\right]\Delta\mu \approx \frac{4}{T_H}\Delta M. \tag{23}$$

From Eqs. (11)–(13) and the mass definition, ΔM can be given by

$$\Delta M = \hbar\omega = 4\pi\hbar G_D T_H \sin\left(\frac{\pi}{D-1}\right). \tag{24}$$

Substituting Eq. (24) into Eq. (23), we can obtain the area spectrum

$$\Delta A = 16\pi\hbar G_D \sin\left(\frac{\pi}{D-1}\right) \tag{25}$$

through a comparison and the corresponding entropy spectrum

$$\Delta S = \frac{\Delta A}{4\hbar} = 4\pi G_D \sin\left(\frac{\pi}{D-1}\right). \tag{26}$$

In summary, with the new physical interpretation for quasinormal modes proposed by Maggiore, we have derived the area and entropy spectra of even D-dimensional large Schwarzschild-AdS black holes from the highly real QNMs, following Kunstatter's method and Maggiore's treatment respectively. It is found that the results from both methods are in full agreement, unlike the cases of rotating black holes.[14,15] The area spectrum and the entropy spectrum are both equally spaced and independent of the black hole parameters. This implies that the area quanta is universal for all types of perturbations, therefore Maggiore's proposal turns out to be very robust in the present case. Moreover, we find that the quantum structure is not related to the cosmological constant, which is consistent with the results of Ref. [17]. The spacing of the area spectrum depends on the dimension of spacetimes, in agreement with the claim in Ref. [16]. However, we observe that the spacing of the entropy spectrum is also related to the spacetime dimension, owing to the proportional relation between the area and the entropy, which is different from Ref. [16]. Our work implies that black holes can be quantized from different asymptotic QNMs not merely from the high damping QNMs, which is usually adopted in the literature. On the other hand, one should keep in mind that since the highly real QNMs, appearing only in even dimensional spaces, have not been found in other black hole spaces, further research is necessary.

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