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Decomposition of Gauge Potential in Ginzburg–Landau Theory *

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Recently, Liu et al. proposed a so-called extended Anderson–Higgs mechanism by studying the (2+1)-dimensional Ginzburg–Landau model in the pseudogap region of high-T_c superconductor (Phys. Rev. B 65 (2002) 132513). We revisit this problem based on a general decomposition of the U(1) gauge potential. Using the bulk superconductor and superconduct ring as examples, we obtain a simpler expression for the extended Anderson–Higgs mechanism. In the former case we indicate that all the phase field can always be "eaten up" by the pure gauge term $\boldsymbol{A}_\parallel$. In the latter case, we decompose the phase field as $\theta(x) = \theta_1(x) + \theta_2(x)$ and find that only the phase field θ_1 connected with Anderson–Higgs mechanism can be canceled by the pure-gauge term $\boldsymbol{A}_\parallel$. On the other hand, the remaining phase field θ_2 connected with $A_\perp$ is multi-valued, which can induce new physical effects such as A-B effect and flux quantization. It is natural to conclude that there is no longitudinal phase fluctuation effect in high-temperature superconductors since longitudinal phase θ_1 is connected with pure-gauge term.

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Because of phase coherence the Ginzburg–Landau (G-L) order parameter for a superconductor can be characterized by $\phi(\boldsymbol{x}) = \sqrt{\rho(\boldsymbol{x})}e^{i\theta(\boldsymbol{x})}$, where $\rho(\boldsymbol{x})$ and $\theta(\boldsymbol{x})$ are real. In this Letter we study the G-L model based on a general decomposition of the U(1) gauge potential. Just as will be shown in the following, for the case of a superconducting ring, the phase $\theta(\boldsymbol{x})$ can be decomposed into two independent nonsingular parts: $\theta_1(\boldsymbol{x})$ and $\theta_2(\boldsymbol{x})$, where θ_1 is connected with the Meissner effect and θ_2 is connected with the topological effect.

The gauge potential $\boldsymbol{A}$ can be divided into the longitudinal field $\boldsymbol{A}_\parallel$ and the transverse field $\boldsymbol{A}_\perp$. In the present work, based on this decomposition, we will give a new expression for the Anderson–Higgs mechanism[1,2] in the G-L theory. It is shown that the gradient of the total phase field can always be canceled by a pure-gauge term $\boldsymbol{A}_\parallel$ in a bulk superconductor. For the case of a superconducting ring, the gradient of a phase field θ_1 can also be removed off by $\boldsymbol{A}_\parallel$ and the remaining multi-value phase field θ_2 is connected with $A_\perp$. If we extend the above conclusion from superconducting ring to pseudogap region in high-temperature superconductors where phase fluctuation effects cannot be neglected. We can explain qualitatively why the transverse phase fluctuations are important and the longitudinal fluctuations are unimportant.

$\boldsymbol{A}_\parallel$ and $\boldsymbol{A}_\perp$ are uniquely obtained through the physical conditions $\nabla \times \boldsymbol{A}_\parallel = 0$, $\nabla \cdot \boldsymbol{A}_\perp = 0$, and the boundary condition $\boldsymbol{A}_\perp(\infty) = 0$. Under U(1) gauge field transformation, we have

$$\boldsymbol{A} \to \boldsymbol{A}' = \boldsymbol{A} + \nabla\Lambda, \tag{1}$$

$$\boldsymbol{A}_\parallel \to \boldsymbol{A}'_\parallel = \boldsymbol{A}_\parallel + \nabla\Lambda, \tag{2}$$

$$\boldsymbol{A}_\perp \to \boldsymbol{A}'_\perp = \boldsymbol{A}_\perp, \tag{3}$$

where Λ is non-singular everywhere. Equations (2) and (3) tell us that the longitudinal part of the vector potential is a pure-gauge part and it transforms in the same manner as does the full $\boldsymbol{A}$, and the transverse field $\boldsymbol{A}_\perp$ is the physical part which is gauge-invariant under all gauge transformations.

Based of the G-L wave function, we first obtain the electric current density describing superconductors:

$$\boldsymbol{j} = \frac{q\rho}{m}\Big(\hbar\nabla\theta - \frac{q}{c}\boldsymbol{A}\Big), \tag{4}$$

where $\boldsymbol{V} = \frac{\hbar}{m}\nabla\theta$ is the usual canonical velocity for electron pairs. Both the phase of the order parameter and gauge potential depend on the gauge chosen, while $\boldsymbol{j}$ is gauge-invariant.

For a bulk superconductor Eq. (4) implies

$$\hbar\nabla\theta = \frac{q}{c}\boldsymbol{A}_\parallel \tag{5}$$

and consequently we deduce the famous London equation

$$\boldsymbol{j} = -\frac{q^2\rho}{mc}\boldsymbol{A}_\perp. \tag{6}$$

The Meissner effect tells us that inside a simply connected superconductor $\boldsymbol{j} = 0$, $\boldsymbol{B} = 0$ and $\boldsymbol{A}_\perp = 0$. In a bulk superconductor we can remove $\boldsymbol{A}_\parallel$ everywhere because of gauge invariance principle. It is clear that a massless Goldstone boson $\theta(x)$ is "eaten up" and the gauge field acquires a mass, which makes the range of force become short (Meissner effect). This is just the

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famous Anderson–Higgs mechanism in quantum field theory.

In Coulomb gauge, the phase field θ must be a constant in a bulk superconductor. However, this result does not hold for a multiply connected superconducting ring, as can be seen from the following discussion.

Here we apply the G-L theory to the case of a superconducting ring. We describe the system using cylindrical polar coordinates with z-axis perpendicular to the plane of the ring. Let us assume that there is a magnetic flux Φ trapped in the superconducting ring. Due to the Aharonov–Bohm (A-B) effect[3] $\boldsymbol{A}_\perp = \frac{\Phi}{2\pi r} \neq 0$ inside the superconductor ring, which differs from the case of a bulk superconductor. Now we make the decomposition $\theta = \theta_1 + \theta_2$, where both the phase field θ_1 and the field θ_2 are nonsingular. In the case of a superconducting ring, Eq. (4) also implies

$$\hbar\nabla\theta_1 = \frac{q}{c}\boldsymbol{A}_\parallel. \tag{7}$$

Thus Eq. (4) becomes

$$\boldsymbol{j} = \frac{q\rho}{m}\left(\hbar\nabla\theta_2 - \frac{q}{c}\boldsymbol{A}_\perp\right), \tag{8}$$

where the phase fields θ_1 and θ_2 are independent of each other, and the Goldstone boson field θ_1 becomes a constant in Coulomb gauge. The multi-valued phase field θ_2 is connected with some topological effects. The form of decomposition of phase field can also be seen in the Kosterlitz–Thouless (K-T) transition.[4] In that case, θ_1 is the nonsingular spin-wave component, and θ_2 represents the singular vortex component. Here we note that there is no K-T transition in a bulk superconductor or a superconducting ring, because K-T transition only occurs in 2D space. However, the K-T transition can occur in the pseudogap region of high-temperature superconductors, where the nonsingular phase field θ_1 can be moved away by the so-called extended Anderson–Higgs mechanism.[5]

Because of the Meissner effect, inside the body of the superconducting ring the current density $\boldsymbol{j} = 0$. Therefore,

$$\hbar\nabla\theta_2 = \frac{q}{c}\boldsymbol{A}_\perp. \tag{9}$$

Now we take the line integral of $\boldsymbol{A}_\perp$ along a curve Γ around the ring:

$$\oint_\Gamma \nabla\theta_2 \cdot d\boldsymbol{l} = \frac{q}{\hbar c}\oint \boldsymbol{A}_\perp \cdot d\boldsymbol{l} = 2\pi\frac{\Phi}{\Phi_0}. \tag{10}$$

From Eq. (10) one sees that the flux trapped in the superconducting ring is quantized by the single-valuedness condition of the wave function $\phi(r,\theta,z) = \phi(r,\theta+2n\pi,z)$, where n is interpreted as the winding number of the wave function. In fact, in the superconducting state, a persistent current will flow around the ring to maintain the constant flux $\Phi = 2n\Phi_0$ in the ring ($\Phi_0 = hc/e$ is the flux quantum).

Now we rewrite Eq. (4) by the Maxwell equation in the static case:

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j}. \tag{11}$$

Thus we obtain the relation

$$\boldsymbol{A} + \lambda^2\nabla \times (\bigtriangledown \times \boldsymbol{A}) = \frac{mc}{q}\boldsymbol{V}, \tag{12}$$

where the constant $\lambda = \sqrt{\frac{mc^2}{4\pi\rho q^2}}$ is called the penetration depth of the magnetic field. In Coulomb gauge, from Eq. (12) we finally obtain the important equation

$$\boldsymbol{A}_\perp - \lambda^2 \bigtriangledown^2 \boldsymbol{A}_\perp = \frac{\Phi_0\nabla\theta_2(x)}{2\pi}, \tag{13}$$

where the physical part of the gauge potential can be represented as $\boldsymbol{A}_\perp = \boldsymbol{A}_{1\perp} + \boldsymbol{A}_{2\perp}$. It is clear that $\boldsymbol{A}_{1\perp}$ is a solution to the homogeneous equation corresponding to Eq. (13), which reflects the Meissner effect (for example, the vector potential must decrease exponentially inside the superconducting ring and $\boldsymbol{A}_{1\perp}(x) = A_{1\perp}(x=0)e^{-\frac{x}{\lambda}}$ for 1D case), while $\boldsymbol{A}_{2\perp}$ is a special solution to the inhomogeneous Eq. (13). It is clear that inside the superconducting ring $\boldsymbol{B} = 0$ and $\boldsymbol{A}_{1\perp} = 0$ because of the Meissner effect. However, $\boldsymbol{A}_{2\perp} \neq 0$ because of A-B effect. In the case of the superconducting ring, it is shown that the Meissner effect and flux quantization are closely related and phase coherence is the origin for these two phenomena. This agrees with the opinion of Ref. [6]. In that reference the authors obtain the same conclusion from the assumptions of off-diagonal long range order (ODLRO).

Finally we work in the pseudogap region of underdoped cuprates superconductors. In high-temperature superconductors the phase stiffness is small and phase fluctuation is important.[7] Franz and Millis[8] considered the effect of classical phase fluctuations on the quasiparticles spectra of underdoped high-temperature cuprate superconductors in the pseudogap region above the superconducting critical temperature T_c. By analyzing the photoemission and tunneling spectroscopy data they show that transverse phase fluctuations are important in the pseudogap region, while longitudinal fluctuations are relatively unimportant at all temperatures. This can be easily understood if we extend Eq. (7) from superconducting ring to pseudogap region. Since the longitudinal field θ_1 is always connected with nonphysical part $\boldsymbol{A}_\parallel$, longitudinal phase fluctuations can never produce any physical effect.

In summary, we have discussed the decomposition of U(1) gauge potential in the G-L model. From Eqs. (5) and (7) we find that a non-singular part of the phase field can always be canceled by the pure-gauge $\boldsymbol{A}_\parallel$ term everywhere. At the same time the "photon" becomes massive and the range of force becomes short, which leads to the Meissner effect. Thus, based on

the decomposition of gauge potential, we give a new expression for the Anderson–Higgs mechanism in the G-L model. On the other hand, the remaining part of the phase is non-singular and multi-valued, which can produce new physical effects, for example, the flux quantization inside a superconducting ring and A-B effect. It should be emphasized that we arrive at this conclusion without using any particular hypotheses, so the qualitative result obtained here can also be applied to the case of high-temperature superconductors.

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