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Transmission Properties of One-Dimensional Photonic Crystals Containing Anisotropic Metamaterials *

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The transmission properties of one-dimensional photonic crystals (1DPCs) containing anisotropic metamaterials are theoretically studied. It is shown that the 1DPCs can possess a similar zero average index (zero-$\bar{n}$) gaps, the edges of zero-$\bar{n}$ gap are weakly dependent on the incident angles, scale length and the polarization of the electromagnetic wave. When an impurity is introduced, a defect mode appears inside the zero-$\bar{n}$ gap with a very weak dependence on incident angles and scaling. It is found that in such photonic crystals, a transmitted Gaussian pulse with its carrier frequency lying in the lower gap edge, in the defect mode and in the bandgap, can experience a positive or negative group delay and hence a subluminal, ultraslow or superluminal propagation with small distortions. These properties of the photonic crystals have potential applications in the transfer of information.

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Photonic crystals are artificial dielectric or metallic periodical structures and they have been intensively studied due to their novel electromagnetic wave characteristics and important scientific and engineering applications.[1,2] Recently, exploitation of photonic crystals containing metamaterials has also received special attention. Metamaterials are usually referred to as left-handed materials, proposed by Veselago in 1968.[3] Because of their simultaneously negative permittivity ε and permeability μ, metamaterials have abnormal electromagnetic behaviors, such as a left-handed set of electric field vector $\boldsymbol{E}$, magnetic field vector $\boldsymbol{H}$ and wave vector $\boldsymbol{k}$ for an electromagnetic wave. Left-handed materials do not exist in nature, but artificial left-handed materials have been realized by using periodic structures either with the unit cell of split-ring resonators and conducting wires[4] or with the unit cell of a left-handed transmission line.[5]

It has been shown that a one-dimensional photonic crystal consisting of isotropic left-handed material and right-handed material possesses a special band gap, corresponding to the zero averaged refractive index, called the zero-$\bar{n}$ gap.[6–8] This band gap is independent of the incident angles and polarization of light. It is also invariant upon the changes of scale length. The metamaterials (interlacing wires and split-ring resonators) investigated in experiments are strongly anisotropic, that is, they are so-called indefinite metamaterials (IMMs), and have recently received increasing attention.[9–13] IMMs have permittivity $\boldsymbol{\varepsilon}$ and permeability $\boldsymbol{\mu}$ tensors in which not all of the components have the same sign. Since only a part of the constitutive parameters are required to be negative, compared with isotropic left-handed materials, IMMs are much easier to fabricate. Various properties and applications of IMMs, such as the characteristics of surface modes and Goos-Hänchen effects have been studied.[10,11] Schurig and Smith constructed high-pass, low-pass, and band-pass spatial filters by using the compensating bilayers of IMMs.[12] In addition, the subluminal or superluminal propagation of a light beam transmitting through IMMs is also investigated.[13]

Research into group delay might be helpful for the development of delay lines and the control of the dispersive effects for systems. However, up to now, most studies on group delay properties have been focused on metamaterials with negative refractive index or conventional periodic structures.[14,15] In this Letter, the transmission properties of one-dimensional photonic crystals composed of isotropic right-handed material and IMMs are investigated. The results show that the zero-$\bar{n}$ gap and the defect mode are weakly dependent on the scaling and incidence angles. Furthermore, the subluminal and superluminal propagation properties are also demonstrated by the temporal evolution of a Gaussian pulse transmitting through a one-dimensional photonic crystal with different carrier frequencies.

Consider the incident Gaussian pulse composed of TE or TM plane waves and the one-dimensional photonic crystal with periodic structure of $(AB)^N$ in air background as shown in Fig. 1. We choose the layers to be parallel to the $x-y$ plane and the z-axis normal to the interfaces of the layers. The layer A is an isotropic right-handed medium, B is an IMM and N is the number of periods. The thicknesses of the two layers are assumed to be d_A and d_B respectively, so

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the lattice constant is $d = d_A + d_B$. For simplicity, we assume that for the IMM layer, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ tensors are simultaneously diagonalizable in the forms:

$$\varepsilon_B = \begin{pmatrix} \varepsilon_{Bx} & 0 & 0 \\ 0 & \varepsilon_{By} & 0 \\ 0 & 0 & \varepsilon_{Bz} \end{pmatrix},$$
$$\mu_B = \begin{pmatrix} \mu_{Bx} & 0 & 0 \\ 0 & \mu_{By} & 0 \\ 0 & 0 & \mu_{Bz} \end{pmatrix}. \quad (1)$$

Without loss of generality, we consider an oblique monochromatic incident wave propagating along the z axis at an incident angle θ with the electric field (magnetic field) polarized along the y axis for TE (TM) waves. For TE mode, the electric and magnetic fields can be expressed as

$$E_{By} = e^{ik_x x}(Ae^{ik_{Bz}z} + Be^{-ik_{Bz}z}),$$
$$H_{Bx} = \frac{-k_{Bz}}{\omega\mu_0\mu_{Bx}} e^{ik_x x}(Ae^{ik_{Bz}z} - Be^{-ik_{Bz}z}),$$
$$H_{Bz} = \frac{k_x}{\omega\mu_0\mu_{Bz}} e^{ik_x x}(Ae^{ik_{Bz}z} + Be^{-ik_{Bz}z}), \quad (2)$$

in the IMM layer, and

$$E_{Ay} = e^{ik_x x}(Ce^{ik_{Az}z} + De^{-ik_{Az}z}),$$
$$H_{Ax} = \frac{-k_{Az}}{\omega\mu_0\mu_A} e^{ik_x x}(Ce^{ik_{Az}z} - De^{-ik_{Az}z}),$$
$$H_{Az} = \frac{k_x}{\omega\mu_0\mu_A} e^{ik_x x}(Ce^{ik_{Az}z} + De^{-ik_{Az}z}), \quad (3)$$

in the right-handed material layer, where $k_{Bz}^2 = \omega^2\varepsilon_{By}\mu_{Bx}/c^2 - \mu_{Bx}/\mu_{Bz}k_x^2$ and $k_{Az}^2 = \omega^2\varepsilon_A\mu_A/c^2 - k_x^2$, k_{Bz} and k_{Az} are the z-components of the wave vector in the IMM and right-handed material layers, k_x is the x-component. A, B, C and D are the four constants which can be determined by the boundary conditions at the interfaces.

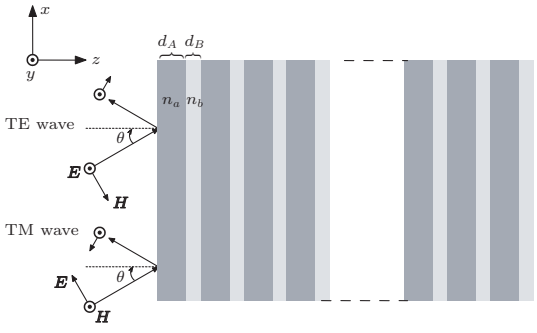

Fig. 1. Schematic diagram of one-dimensional photonic crystal consisting of alternating the right-handed material (A) and IMM (B) under any incident angle θ for TE and TM waves.

For the TE mode case, based on Bloch's theorem and the boundary condition, the dispersion relation for any incident angle follows[16]

$$\cos(qd) = \cos(k_{Az}d_A)\cos k_{Bz}d_B) - \frac{1}{2}\left(\frac{k_{Bz}\mu_A}{k_{Az}\mu_{Bx}} + \frac{k_{Az}\mu_{Bx}}{k_{Bz}\mu_A}\right)\sin(k_{Az}d_A)\sin(k_{Bz}d_B), \quad (4)$$

where q is the z-component of the Bloch wave vector. Similarly we can obtain the dispersion relation for the TM mode. When the wave vectors and the thickness of the layers satisfy the condition

$$k_{Az}d_A + k_{Bz}d_B = m\pi, \quad (m \in \text{integers}), \quad (5)$$

the Bloch wave vector component q may be imaginary, indicating the presence of band gaps, where wave propagation is inhibited. Especially when $m = 0$, the zero-$\bar{n}$ gap is obtained, which is quite different from the conventional Bragg gap. For a finite photonic crystal $(AB)^N$, the electric and magnetic fields at any two positions z and $z+\Delta z$ in the same layer can be related by a transfer matrix[17]

$$T_{A,B} = \begin{pmatrix} \cos(k_{A,Bz}d_{A,B}) & \mathcal{X}_{12} \\ \mathcal{X}_{21} & \cos(k_{A,Bz}d_{A,B}) \end{pmatrix}. \quad (6)$$
$$\mathcal{X}_{12} = i\frac{\mu_{A,Bx}\omega}{k_{A,Bz}c}\sin(k_{A,Bz}d_{A,B})$$
$$\mathcal{X}_{21} = i\frac{k_{A,Bz}c}{\mu_{A,Bx}\omega}\sin(k_{A,Bz}d_{A,B})$$

With the transfer matrix method,[16,17] we can obtain the transmission coefficient of a monochromatic plane wave,

$$t(\omega) = 2p_0\{[p_0x_{22}(\omega) + p_sx_{11}(\omega)] - [p_0p_sx_{12}(\omega) + x_{21}(\omega)]\}^{-1}, \quad (7)$$

where $p_0 = p_s = \cos\theta$ for the air background in the space $z < 0$ and $z > Nd$, x_{ij} $(i, j = 1, 2)$ are the matrix elements of $X_N(\omega) = \prod T_{A,B}$, which represents the total transfer matrix connecting the fields at the incident end and the exit end.

In our calculation, for the TE wave, we choose $\varepsilon_{By} = 4 - 20/f^2$, $\mu_{Bx} = 1 - 20/f^2$, $\mu_{Bz} = 2$. For the TM wave, we have $\varepsilon_{Bx} = 4 - 20/f^2$, $\mu_{By} = 1 - 20/f^2$, $\varepsilon_{Bz} = 4$, where f is the frequency measured in units of GHz. The permittivity and permeability of the isotropic right-handed material layer are chosen to be $\varepsilon_A = 2$, $\mu_A = 1$. We have $d_A = 12\,\text{mm}$, $d_B = 6\,\text{mm}$ and $N = 10$. By studying the optical transmission spectra of the one-dimensional photonic crystals, we analyze the dependence of the photonic band gaps on the incident angles and scale lengths. In Fig. 2, we show the transmission as a function of frequency under different incident angles. It is illustrated that the Bragg gap shifts towards high frequency for both the TE and TM modes as the incident angle increases. However, a special gap appears at around 2.0 GHz, which should be the zero-$\bar{n}$gap since the condition

of $\bar{n} = (n_{Az}d_A + n_{Bz}d_B)/(d_A + d_B) = 0$ is satisfied according to the above parameters, where $n_{Az}^2 = \varepsilon_A\mu_A - \sin^2\theta$ and $n_{Bz}^2 = \varepsilon_{By}\mu_{Bx} - \mu_{Bx}/\mu_{Bz}\sin^2\theta$. It can be found that the upper edge of the zero-$\bar{n}$ gap for the TE mode and the lower edge of the zero-$\bar{n}$ gap for the TM mode almost do not shift when the incident angle increases. The zero-$\bar{n}$ gap is insensitive to the variation of the incident angles of the wave compared with the Bragg gaps.

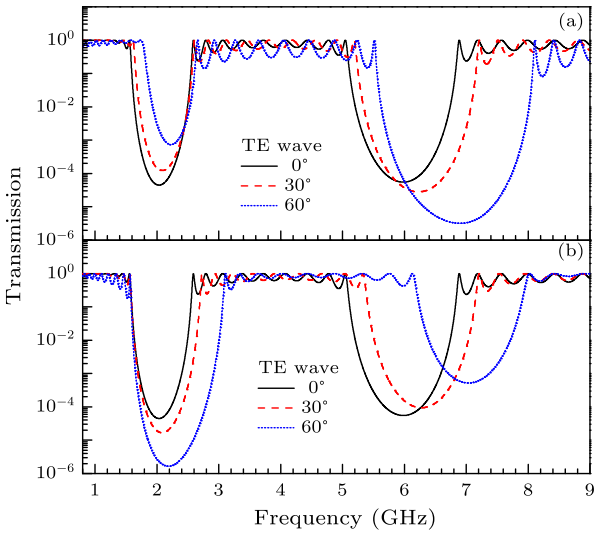

Fig. 2. Transmissions of one-dimensional photonic crystals for TE (a) and TM (b) waves under different incident angles with $d_A = 12\,\text{mm}$, $d_B = 6\,\text{mm}$ and $N = 10$.

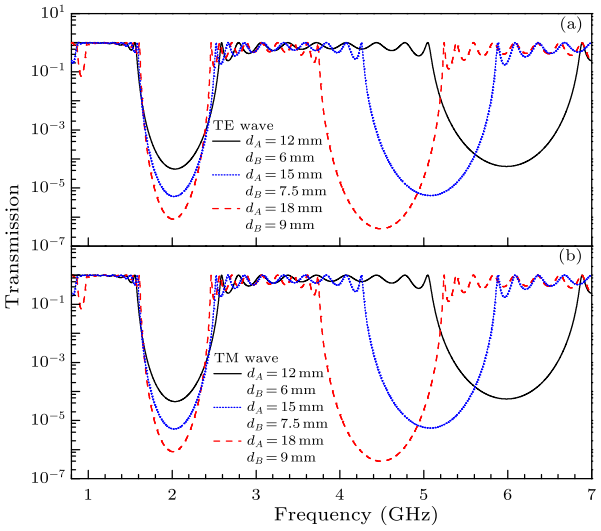

Fig. 3. Transmissions of one-dimensional photonic crystals for TE (a) and TM (b) waves at different scale lengths: $d_A = 12\,\text{mm}$, $d_B = 6\,\text{mm}$ (solid line); $d_A = 15\,\text{mm}$, $d_B = 7.5\,\text{mm}$ (dotted line); $d_A = 18\,\text{mm}$ and $d_B = 9\,\text{mm}$ (dashed line), and $N = 10$ with normal incidence.

In Fig. 3, we demonstrate the dependence of the transmissions on the scale lengths of the one-dimensional photonic crystals. We fit the ratio of $d_A/d_B = 2$ and increase the length from 12 mm to 18 mm for the layer A. With the changing scale lengths, not only does the center of the zero-$\bar{n}$ gap at 2.0 GHz not move, but also the two gap edges remain at nearly the same frequencies. However, the spectra of the Bragg gaps are significantly affected by the scaling. Next we consider the case in which the localized defect modes are induced by a defect layer inside the one-dimensional photonic crystal with the structure of $(AB)^{N/2}Z(AB)^{N/2}$. We choose a Z material as the defect layer with a refractive index $n_z = 2.5$ and a thickness $d_z = 24\,\text{mm}$. In Fig. 4, it explicitly reveals that the spectral position of the defect modes inside the zero-$\bar{n}$gap (at about 1.85 GHz) almost does not move, presenting a very weak dependence on the incident angles. However, the defect modes inside the Bragg gaps are strongly affected when the incident angle increases from 0° to 45°. In addition, it is also found that the defect modes in the zero-$\bar{n}$ gap are insensitive to a change of the scale length in our calculations.

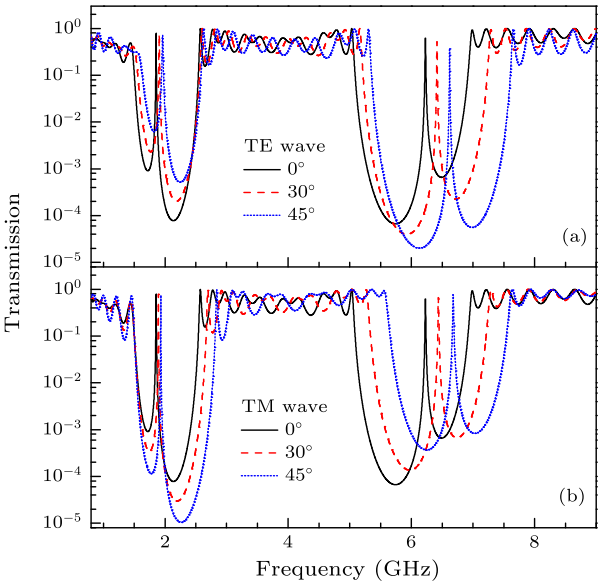

Fig. 4. Transmissions of one-dimensional photonic crystals with defect mode for TE (a) and TM (b) waves at different incident angles.

To investigate the subluminal and superluminal propagation properties through the one-dimensional photonic crystal containing IMMs, we analyze the temporal evolution of a Gaussian pulse at the exit end. The electric field of the Gaussian pulse at the incident surface is expressed as

$$E_i(t) = E_0 \exp\Big(-\frac{t^2}{\tau^2}\Big), \tag{8}$$

where E_0 is a constant, its Fourier spectrum is

$$E_i(\omega) = \frac{E_0\tau}{2\sqrt{\pi}} \exp\Big(-\frac{\tau^2(\omega-\omega_0)^2}{4}\Big), \tag{9}$$

and the transmitted electric field is

$$E_t(t) = \int t(\omega)E_i(\omega)\exp(-i\omega t)d\omega, \tag{10}$$

where ω_0 is the carrier frequency and τ is the characterized time of the pulse. The integral of the different

Fourier components of the Gaussian pulse, which do not have the same phase factor, might result in the advance or delay of the Gaussian peak at the exit end.

In Fig. 5, we plot the temporal evolution of the TE polarized Gaussian pulse ($\tau = 20$ ns) through the one-dimensional photonic crystal (1DPC) ((Fig. 2(a), the solid line) with the total length ($L = 180$ mm). For comparison, we have also shown the reference pulse, i.e., a pulse that travels the distance L in free space. The transmitted electric fields are normalized. The Gaussian pulse with the carrier frequency at 2 GHz, which is inside the zero-$\bar{n}$ gap, propagates faster in 1DPC than in air. It has about 0.3 ns time advance due to the anomalous dispersion within the band gap, which indicates the superluminal propagation. When we change the carrier frequency to 1.6 GHz, which is the lower zero-$\bar{n}$ gap edge, the Gaussian pulse propagates at a subluminal speed because the group index is more than unity as shown in Ref. [18], and the pulse delay by about 3 ns. These properties of the group delay can be further understood in terms of the magnitude and phase of the transmission coefficient. Moreover, the transmitted pulse keeps the Gauss form with negligible distortion.

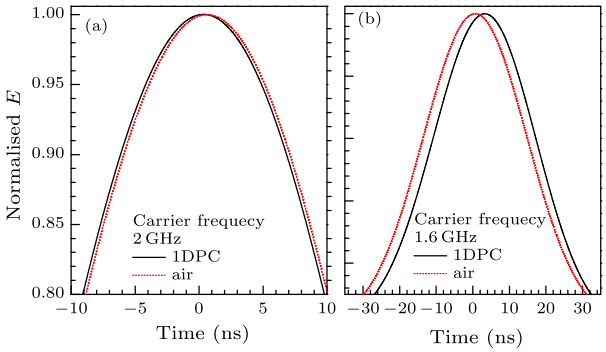

Fig. 5. Temporal evolution of a TE mode Gaussian pulse through the one-dimensional photonic crystal (1DPC) or air with the carrier frequencies of 2 GHz (a) and 1.6 GHz (b), respectively.

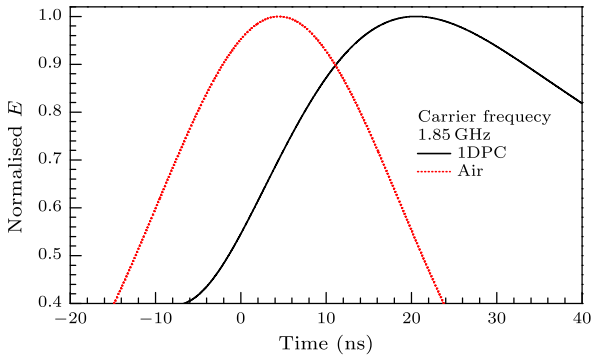

Fig. 6. Temporal evolution of a TE mode Gaussian pulse through the 1DPC with defect mode and air. The carrier frequency is 1.85 GHz.

The temporal evolution of the Gaussian pulse transmitted through a defective photonic crystal (Fig. 4(a), the solid line) is also studied as shown in Fig. 6. The Gaussian pulse with 1.85 GHz carrier frequency propagates at an ultraslow speed compared with the case in air, there is about 16 ns time delay therefore to slow light due to the steep changes in phase and in the effective refractive index. It also illustrates that the Gaussian pulse has some deviation in comparison with the incident one when it passes through the defective structure. That is because the defect mode pick is rather narrow compared with the frequency spectrum width of the pulse. It is worth mentioning that if the material dispersion is anomalous within the considered frequency band, the group velocity is strongly different from energy velocity and the energy transport cannot be definitely illustrated by group velocity.[19]

In conclusion, we have studied the properties of the zero-$\bar{n}$ gap and the defect mode of the 1DPC containing IMMS. The results show that the zero-$\bar{n}$ gap and the defect mode weakly depend on the incident angles and the scale lengths. The superluminal and subluminal propagations of the PC are illustrated by the temporal evolution of a Gaussian pulse with different carrier frequencies. Especially at the defect mode, the group speed is ultraslow. Such properties of photonic crystals have potential applications in the propagation of information and delay systems. In addition, the study of the structure containing anisotropic metamaterial in terms of loss, quasi-periodic succession and disorder remains a topic for further studies, some of which are now under investigation.

References

[1] Yablonovitch E 1987 *Phys. Rev. Lett.* **58** 2059
John S 1987 *Phys. Rev. Lett.* **58** 2486
[2] Joannopoulos J, Meade R and Winn J 1995 *Photonic Crystals: Molding the Flow of Light* (Princeton: Princeton University)
[3] Veselago V G 1968 *Sov. Phys. Usp.* **10** 509
[4] Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 77
Smith D R et al 2000 *Phys. Rev. Lett.* **84** 4184
[5] Caloz C and Itoh T 2006 *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications* (New York: Wiley & Sons)
Si L M, Sun H J et al 2010 *Chin. Phys. Lett.* **27** 034106
[6] Li J, Zhou L, Chan C T and Sheng P 2003 *Phys. Rev. Lett.* **90** 083901
[7] Bria D et al 2004 *Phys. Rev.* E **69** 066613
[8] Jiang H T et al 2003 *Appl. Phys. Lett.* **83** 5386
[9] Smith D R and Schurig D 2003 *Phys. Rev. Lett.* **90** 077405
[10] Yan W et al 2007 *J. Opt. Soc. Am.* A **24** 530
[11] Cheng M and Chen R 2009 *Chin. Phys. Lett.* **26** 014101
[12] Schurig D and Smith D R 2003 *Appl. Phys. Lett.* **82** 2215
[13] Xu G D, Su L, Pan T, Zang T C 2008 *Physica* B **403** 3417
[14] Ziolkowski R W 2001 *Phys. Rev.* E **63** 046604
Dutta Gupta S et al 2004 *Phys. Rev.* B **69** 113104
[15] Wang L G et al 2004 *Phys. Rev.* E **70** 016601
[16] Wang L G et al 2004 *Phys. Rev.* B **70** 245102
[17] Wang S and Gao L 2005 *Eur. Phys. J.* B **48** 29
[18] Solli D R et al 2003 *IEEE J. Sel. Top. Quantum Electron.* **9** 40
[19] Zhang K Q and Li D J 1994 *Electromagnetic Theory in Microwaves and Optoelectronics* (Beijing: Publishing House of Electronic Industry) p 466 (in Chinese)