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Improvement of a Fission-Like Model for Nuclear α Decay *

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The α decay constant is the product of the penetrability P and assault frequency ν_0 in the fission-like model. An effective assault frequency P_ν replacing the previous assault frequency ν_0 is introduced for improvement of a fission-like model named the generalized liquid drop model (GLDM) to describe the nuclear α decay process more accurately. Two analytical formulae are proposed for the effective assault frequency due to experimental data within the GLDM. The improved model can be used to give accurate calculations for α decay half-lives.

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Shortly after the advent of quantum mechanics, Gurney and Condon[1] and Gamow[2] simultaneously explained the α decay process in terms of the quantum tunneling effect. Since then many theoretical approaches have been proposed to describe the α decay phenomenon, such as the Viola–Seaborg rule,[3] the cluster model (CM),[4−11] the generalized liquid drop model (GLDM),[12−17] and the density-dependent M3Y (DDM3Y) effective interaction.[18,19] The Viola–Seaborg formula is the simplest way to estimate the α decay half-lives. Buck *et al.*[4] used the CM with the phenomenological cosh potential to calculate the decay half-lives of many α emitters and cluster emitters; the results agree reasonably well with the experimental data, the method was improved by the group of Ren for deformed nuclei and systematical studies have been executed recently.[5−9] The GLDM, which includes a proximity energy term, has successfully allowed the description of fusion, fission, heavy cluster emission, α decay, and proton radioactivity.[20] In the framework DDM3Y, the nuclear potentials have been obtained microscopically by double folding the α and daughter nuclei density distributions with a realistic M3Y effective nucleon-nucleon interaction, and it is used to describe the fusion, α decay and heavy-ion collision process.

In fact, the above approaches can be classified into two types of method: the cluster-like theory and the fission-like theory. The cluster-like theories suppose that the α particle is preformed in the mother nucleus with a certain performance factor while the fission-like approaches consider that the α particle is formed progressively during the very asymmetric fission of the parent nucleus. The experimental investigation cannot unambiguously distinguish these two formation modes. However, the possible one-body configurations play a minor role because in the quasi-molecular decay path investigated in the α decay process the potential barrier is governed by the balance between the repulsive Coulomb forces and the attractive proximity forces, consequently the barrier top is more external and lower than the pure Coulomb barrier and corresponds to two separated fragments. The difference between the two approaches appears mainly in the way the decay constant is determined. In the preformed cluster models, the decay constant λ is the product of the preformed factor P_α, the constant assault frequency ν_0 and the barrier penetrability P. However, in the fission-like models, the α decay constant is the product of the penetrability P and assault frequency ν_0. The potential barrier including the Coulomb repulsion, nuclear attraction due to proximity potential, and rotational energy due to the angular momentum transfer of the α particle plays the decisive role in the WKB penetration probability, which is constructed by the GLDM in the present study. For a deformed nucleus, the macroscopic GLDM energy is defined as[13]

$$E = E_V + E_C + E_S + E_{\mathrm{Prox}}, \tag{1}$$

where the volume E_V, surface E_S, Coulomb E_C and proximity E_{Prox} energy are introduced to take into account the additional surface effects due to the attractive nuclear forces between the surfaces in a neck or a gap between two separated fragments:

$$E_{\mathrm{Prox}} = 2\gamma \int_{h_{\min}}^{h_{\max}} \Phi[D\,(r,h)\,/b]2\pi h dh, \tag{2}$$

where h is the distance varying from the neck radius or zero to the height of the neck border, D is the distance between the surfaces in regard and $b = 0.99\,\mathrm{fm}$

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is the surface width, Φ is the proximity function of Feldmeier[12] and the surface parameter γ is the geometric mean between the surface parameters of the two nuclei or fragments. The barrier penetrability P is calculated within the action integral,

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_{\rm in}}^{R_{\rm out}}\sqrt{2B(r)(E(r)-E(\text{sphere}))}dr\right]. \quad (3)$$

Two approximations have been used: $R_{\rm in} = R_d + R_\alpha$ and $B(r) = \mu$, where R_d and R_α are the respective radii of the daughter nuclei and the α particle, and μ is the reduced mass. $R_{\rm out}$ is simply determined by the Coulomb potential. The α decay constant is defined as

$$\lambda = \nu_0 P. \quad (4)$$

The α decay half-life $T_{1/2}$ is calculated as

$$T_{1/2} = \ln 2/\lambda. \quad (5)$$

In the previous study on α decay with the GLDM,[12,13] the assault frequency is fixed as ν_0=$10^{20}s^{-1}$. The theoretical data are slightly higher than the experimental ones for lighter nuclei and systematically lower for the heaviest systems.[12,13] Thus we suggest that the assault frequency should be adjusted to give a accurate estimation for the α decay half-lives. In fact, the assault frequency is also introduced in DDM3Y, whose values are nuclear α decay energy dependent.[21] Thus we would like to introduce a new assault frequency P_ν named the effective assault frequency, which may be deduced from the penetration probability P which is calculated within the WKB approximation and the experimental α decay half-life $T_{1/2}^{\rm expt}$ by

$$P_\nu = \frac{\ln 2}{PT_{1/2}^{\rm expt}}. \quad (6)$$

In this work, the values of P_ν for 154 even-even nuclei are extracted using the above formula. Figure 1 displays a plot of $\log_{10} P_\nu$ as a function of the neutron numbers. It is evident that the values of P_ν present a sharp decrease about the neutron magic number $N = 126$, reconfirming this neutron closure shell structure. When the neutron number is beyond the region about 160, the decreased trend appears again, which gives us some signals for the island of stability of superheavy nuclei, and more experimental α decay data are desired in this region to give sufficient proof for this island.

In order to see the correlation between the effective assault frequency and the structure properties more clearly, the effective assault frequencies of Po, Rn, Ra, and Th isotopes are given as a function of the neutron number N in Fig. 2. The effective assault frequencies of the isotopes generally decrease with increasing neutron number up to the spherical shell closure $N = 126$, where the minimum values of P_ν occur, and then they increase quickly with neutron number.

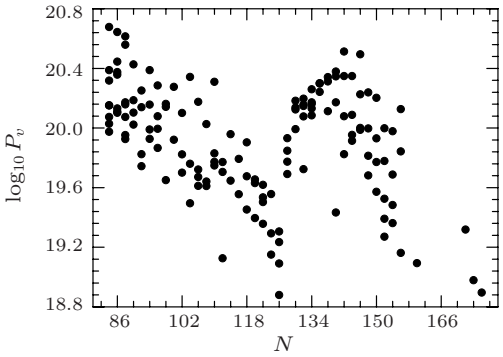

Fig. 1. Behavior of $\log_{10} P_\nu$ for 154 even-even nuclei versus the neutron number N.

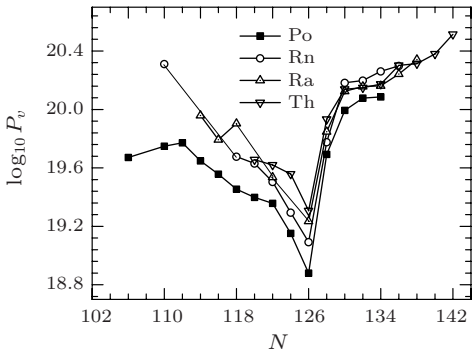

Fig. 2. Characteristics of $\log_{10} P_\nu$ for the even-even isotopes of Po, Rn, Ra and Th.

If we pay more attention to Fig. 2 for $N = 126$ isotones, the values of the effective assault frequency increase when the proton number goes away from the magic number $Z = 82$ (from ^{212}Po to ^{218}Th); this characteristic also appears in most cases for the other isotones. As a conclusion the dependence of effective assault frequencies on the nucleon numbers can provide useful information on nuclear structure. The closer the nucleon number is to the magic numbers, the smaller the effective assault frequency of the α cluster inside the mother nucleus will be. Therefore a formula which contains the nuclear shell effects can be reasonably proposed:

$$\log_{10} P_\nu = 24.806 - 0.054Z + 0.018(N-126) - \frac{0.233}{(N-126+0.5)^2}. \quad (7)$$

The results are plotted in Fig. 3, and it turns out that the extracted effective assault frequency can be well reproduced by Eq. (7). The standard deviation is defined as

$$\sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{n}\frac{[\log_{10}(P_\nu^{{\rm expt},i}/P_\nu^{{\rm fit},i})]^2}{n}}. \quad (8)$$

The standard deviation obtained from Eq. (7) is

0.2741, which means that the average deviation between the theoretical estimates and experimental data for α decay half-lives will be $10^{0.2741} = 1.88$, if the previous assault frequency $\nu = 10^{20}$ is used to replace the values from this formula within the GLDM.

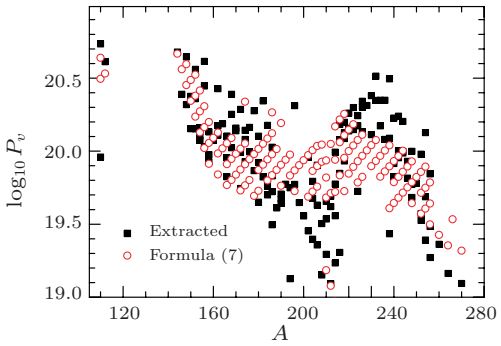

Fig. 3. (Color online) Extracted (black squares) and fitted $\log_{10} P_\nu$ from Eq. (7) (circles) for 154 even-even nuclei.

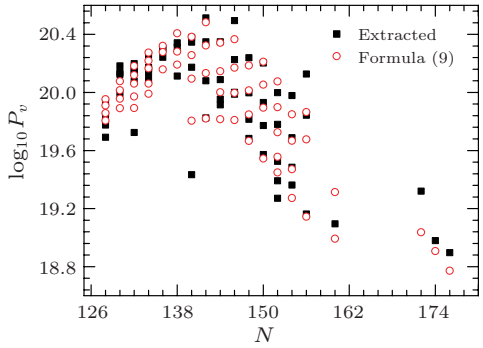

Fig. 4. (Color online) Extracted (black squares) and fitted $\log_{10} P_\nu$ from Eq. (9) (circles) for even-even heavy nuclei ($Z > 82$ and $N > 126$).

Finally, a very exact formula for the effective assault frequency as a function of the charge number Z, the mass number A of the parent nucleus, and the experimental α decay energy Q is proposed for the heavy even-even nuclei ($Z > 82$ and $N > 126$),

$$\log_{10} P_\nu = 28.087 + 0.2675Z^{0.5} - 0.5447A^{0.5} - 0.8336Q^{0.5}. \quad (9)$$

The standard deviation between the extracted data and the values obtained from Eq. (9) is only 0.1622, implying that the average deviation between the theoretical estimates and the experimental data for α decay half-lives will be $10^{0.1622} = 1.45$. Using this formula within the GLDM, the experimental data will be well reproduced and it is reliable to predict the α decay half-lives of even-even superheavy nuclei. The $\log_{10} P_\nu$ calculated by Eq. (9) and extracted by Eq. (6) are shown in Fig. 4. It is clear that the extracted data and the results from Eq. (9) coincide with each other nearly exactly.

In conclusion, an effective assault frequency P_ν is extracted from the experimental data and two formulae are proposed. These results indicate that the known experimental α decay half-lives will be reproduced accurately when the new assault frequency P_ν is calculated by Eq. (7) or Eq. (9) in the framework of the GLDM. The improved GLDM can be used to provide reliable predictions for unobserved heavy and superheavy nuclei supplied with excellent α decay energy, and the work is in progress.

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