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Lagrangian Structure Function's Scaling Exponents in Turbulent Channel Flow *

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The Lagrangian structure function's scaling exponents and intermittency of three-dimensional incompressible turbulent channel flow are investigated by using direct numerical simulation. The intermittency in streamwise velocity increments is found to increase in the near-wall region, which can be attributed to the presence of strong mean shear and organized motions in the near-wall region. It is found that the intermittency of transverse velocity increments is weaker than that of longitudinal ones. The present ESS evaluation of $\zeta_L(q)$ for the structure function of the streamwise velocity component in the channel centre is fairly close to experimental estimates of isotropic turbulence.

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Intermittency is a basic feature of fully developed turbulence.[1] For Eulerian turbulence, intermittency provides corrections to Kolmogorov's scaling law,[2] which depicts a similar relationship between statistical properties of fluctuations at different small scales. The Kolmogorov[2] predictions are based upon assumptions that the statistical properties of the velocity field are locally homogeneous and isotropic and that there exists a constant-energy cascade from large to small scales.

The intermittency can also be investigated in a Lagrangian framework.[3] The knowledge of Lagrangian statistics in a fully developed turbulent flow is also a key ingredient for the development of Lagrangian stochastic models in such diverse contexts as turbulent combustion, pollutant dispersion, cloud formation and industrial mixing. Given the importance of this problem, there are comparatively few experimental studies of turbulent Lagrangian statistics due to the difficulty of tracing the particles in an experiment. However, a recent breakthrough has been made by La Porta *et al.*[4] to track three-dimensional fluid particle trajectories with a relatively high resolution, which enables the detailed study of Lagrangian statistical properties. At the same time, numerical simulation such as large-eddy simulation (LES) and direct numerical simulation (DNS) have also been applied to investigate Lagrangian statistics in homogeneous and isotropic turbulent flows, and also channel flows, and its important contribution has been recently reviewed by Yeung.[5]

In a Lagrangian framework, let $v(y_0,t)$ denote the Lagrangian velocity at time t of the fluid particle originating from y_0 at time $t=0$, the flow is described by the velocity $v(y_0,t)$. It is well recognized that Lagrangian velocity increments $\Delta v(\tau) = |v(y_0,t+\tau)-v(y_0,t)|$, where statistical homogeneity is assumed, are quasi-Gaussian for time lags τ of order T^L (T^L is Lagrangian integral time scale), but become increasingly intermittent at higher frequencies.[6] For the temporal increments of the Lagrangian velocity, Landau and Lifshitz[7] proposed a 1/2 law in 1944. Novikov[3] generalized $\Delta v(\tau)$ with a Lagrangian intermittency framework,

$$\langle \Delta v^q(\tau)\rangle \sim \tau^{\zeta_L(q)}. \tag{1}$$

Neglecting intermittency, we have $\zeta_L(q)=q/2$. In this framework, the third-order moment for the Eulerian velocity is analogous to the second-order moment for the Lagrangian velocity. In the case of intermittency, $\zeta_L(q)$ is nonlinear and convex, and the nonintermittent function is valid only for $q=2$, i.e. $\zeta_L(2)=1$.

Different hypotheses can be invoked to relate Eulerian and Lagrangian statistics and obtain some relations for Eulerian and Lagrangian scaling exponents, see Ref. [8] for more details. From the relation between Lagrangian and Eulerian scaling exponents, we obtain the exponent of the Lagrangian structure function[8,9]

$$\zeta_L(q) = (a-1+\sqrt{(1-a)^2+4qb}-2bq)/2b, \tag{2}$$

where $a=(2+\mu)/6$ and $b=\mu/18$, $\mu=0.23$ is the intermittency parameter.

The lognormal model (Eq. (2)) will be used to compare our DNS results in turbulent channel flow and experimental estimations published in recent papers.[6,10−14]

A direct numerical simulation of turbulent channel flow at a low Reynolds number was performed to

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evaluate the Lagrangian structure function's scaling exponents. The Reynolds number Re_τ is 100, where $Re_\tau = u_\tau\delta/\nu$, and u_τ, δ and ν represent the friction velocity, half width of the channel and kinematic viscosity, respectively. The Reynolds number of the present simulation is at the lowest end of the turbulent channel flow. Near-wall coherent structure such as low-speed streak and quasi-streamwise vortex is yet realized in the simulation and we are particularly interested in the intermittency.

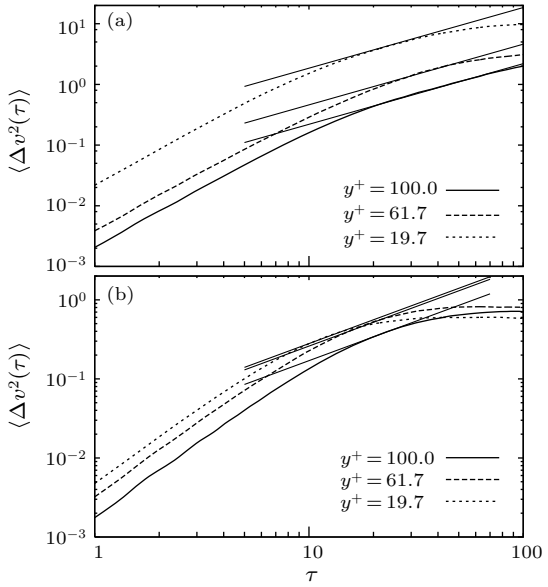

Fig. 1. $\langle\Delta v^2(\tau)\rangle$ vs τ in streamwise (a), wall-normal (b) ($Re_\tau = 100$).

The dependence of scaling exponents on the distance from the wall (or the starting points in the Lagrangian exponents) was also briefly investigated. The velocity fields of $Re_\tau = 100$ computed by DNS were stored at the interval $\Delta t^+ = 0.2$ for a period of 3000 in wall units. Fluid particle velocity along a particle trajectory was computed by employing the 3rd order Hermite polynomials in the homogeneous directions and a Chebyshev polynomial in the wall-normal direction. Periodic boundary conditions were imposed on the streamwise and spanwise directions to obtain the velocity of the fluid particles. The particles were tracked through the computed velocity fields to obtain Lagrangian velocity data.[15−16] The magnitude of the number of samples is 10^6 order, which provides a good estimation for the moments of velocity increments up to 6.

For turbulence at low or moderate Reynolds numbers, there exists no wide enough inertial range to determine the scaling exponents smoothly. To extend the applicability of the scaling law even to the low Reynolds number case, Benzi *et al.*[9] developed an extended self-similarity (ESS) scheme.

The introduction of the ESS allows the study of scaling laws at moderate and low Reynolds number to be performed. In the Eulerian case, the reference structure function is of third order, as it is assumed to scale linearly with the increment. In the Lagrangian case, the second-order structure function plays a similar role and is used as a reference,

$$\langle\Delta v^q(\tau)\rangle \sim \langle\Delta v^2(\tau)\rangle^{\zeta_L(q)} . \tag{3}$$

Using the ESS, an accurate estimate for the scaling exponents ($\zeta_E(q)$ and $\zeta_L(q)$) could be obtained at low Reynolds number. We shall consider the streamwise velocity scaling exponents $\zeta_E(q)$ and scaling exponents $\zeta_L(q)$ of v_i ($i = 1$, 2 and 3). The differences of scaling exponents between different velocity exponents will be addressed later.

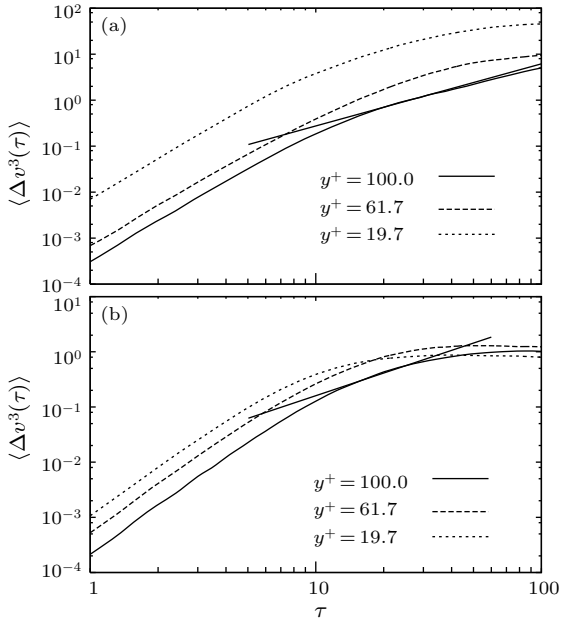

Fig. 2. $\langle\Delta v^3(\tau)\rangle$ vs τ in streamwise (a), wall-normal (b) ($Re_\tau = 100$).

Figures 1 and 2 show the second and third moments of Lagrangian velocity increment ($\langle\Delta v^2(\tau)\rangle$ and $\langle\Delta v^3(\tau)\rangle$) for particles released from $y^+ = 19.7$, 61.7 and 100.0 in the streamwise and wall-normal directions, respectively. From Fig. 2, a rather narrow inertial range (indicated by the thin solid lines with slope 1) is observed for all three initial locations and for the streamwise and wall-normal velocity components. The same result can be seen from Fig. 3 as well. From Fig. 3 using the same range of scales as indicated in Fig. 2, a value of $\zeta_L(3)$ can be estimated, as $\zeta_L(3) \approx 1.36$ (shown by the thin solid line with slope 1.36) for $y^+ = 100.0$. As mentioned above, an ESS method may be applied to evaluate the relative scaling exponents.

Following the ESS method, Figure 3 shows $\langle\Delta v^3(\tau)\rangle$ versus $\langle\Delta v^2(\tau)\rangle$ for particles released from the three different initial positions and for the streamwise and wall-normal velocity components. A strikingly wide scaling range is observed in Fig. 3 and it is

quite easy to determine $\zeta_L(3) = 1.36$ for all the cases. It also clearly shown that the scaling exponent is slightly becoming smaller when the starting point approaches the solid wall. In the same way the exponents $\zeta_L(q)$ for $q = 1, 4, 5, 6$ can be estimated. The corresponding results are plotted in Fig. 3. Here Mordant *et al.*[13] gave the average value of experimental estimates for four isotropic turbulent flows having Reynolds numbers Re_λ from 510 to 1000. Biferale *et al.*[6] estimated the Lagrangian structure function's scaling exponent for the isotropic turbulent flow ($Re_\lambda = 280$) using DNS. The agreement with Eq. (2) is good for low orders of moment. For larger moments ($q \geq 4$), it is clear from Figs. 4(a) and 4(b) that the intermittences in the streamwise and wall-normal directions increase with the decrease of the initial distance from the wall. In the Eulerian case, through the observation of $\zeta_E(q)$, the increase in intermittency streamwise velocity component toward the wall has also been observed by Antonia *et al.*[17] and Amati *et al.*,[18] which is attributed to the presence of strong mean shear and organized motions in the near wall region. In addition, $\zeta_L(q)$ in the central range ($y^+ = 100.0$) almost coincides with the experimental estimate. Finally, it is clearly seen that Eq. (2) agrees well with Biferale *et al.*[6] and is close to experimental and DNS result at $y^+ = 100.0$. Figure 4 also shows that the exponents of streamwise velocity have a greater deviation from the homogeneous and isotropic exponents than the exponents of transverse velocity. This means that the intermittency of longitudinal velocity increments is stronger than that of transverse increments.[19]

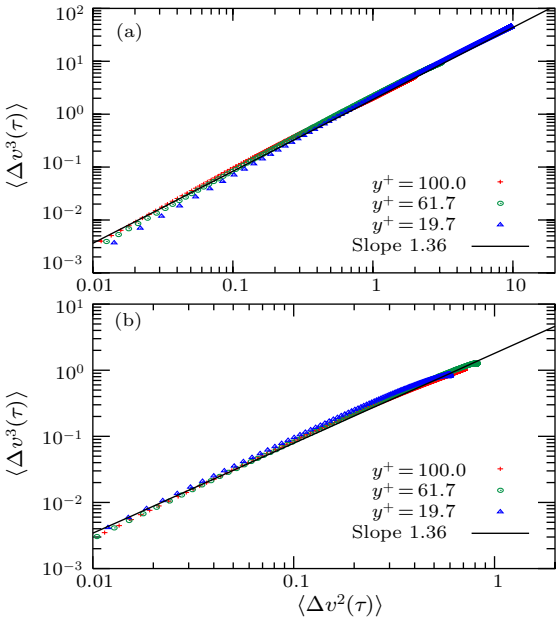

Fig. 3. $\langle \Delta v^3(\tau) \rangle$ vs $\langle \Delta v^2(\tau) \rangle$ for particles released from $y^+ = 19.7$, 61.7 and 100.0 in streamwise (a), wall-normal (b) ($Re_\tau = 100$).

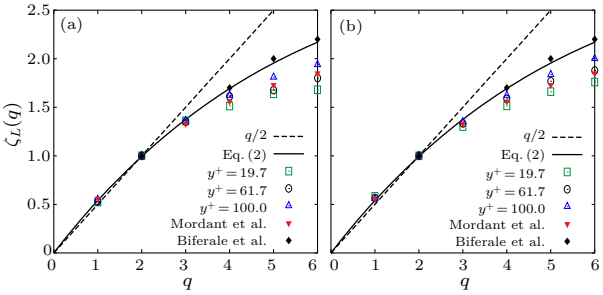

Fig. 4. Comparison of the Lagrangian structure function's scaling exponent $\zeta_L(q)$ for particles released from $y^+ = 19.7$, 61.7 and 100.0 in streamwise (a), wall-normal (b) ($Re_\tau = 100$).

The Lagrangian structure function's scaling exponents are investigated using DNS of turbulent channel flow at a low Reynolds number. The intermittency in streamwise velocity increments is found to increase in the near-wall region, which can be attributed to the presence of strong mean shear and organized motions in the near-wall region. It is found that the intermittency of transverse velocity increments is weaker than that of longitudinal ones. The present ESS evaluation of $\zeta_L(q)$ for the structure function of the streamwise velocity component in the channel centre is fairly close to experimental estimates of isotropic turbulence and a lognormal fit (Eq. (2)).

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