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Chorus-Driven Outer Radiation Belt Electron Dynamics at Different L-Shells *

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Energetic outer radiation belt electron phase space density (PSD) evolution due to interaction with whistler-mode chorus at different L-shells is investigated by solving the diffusion equation including cross diffusion terms. It is found that the difference of diffusion rates for different L-shells occurs primarily at pitch angles 0°–50° and around 90°. In particular, diffusion rates for $L = 6.5$ are found to be 5–10 times larger than that for $L = 3.5$ at these pitch angles. In the presence of cross terms, PSD for ∼ MeV electrons after 24 h decreases by about 25, 12, 10 and 8 times at $L = 3.5$, 4.5, 5.5 and 6.5 near the loss cone, and increases by about 55, 45, 30 and 20 times at larger pitch angles, respectively. After 24 h, the ratios between ∼ MeV electron PSDs from simulations without and with cross diffusion at $L = 3.5$, 4.5, 5.5 and 6.5 are about 350, 600, 800 and 800 near the loss cone, and become 5, 5.5, 6.5 and 8 at pitch angle 90°, respectively. These results demonstrate that neglect of cross diffusion generally results in the overestimate of PSD, and the cross diffusion plays a more significant role in the resonant interaction between chorus waves and outer radiation belt electrons at larger L.

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The Earth's outer radiation belt electron flux is found to exhibit a significant variability during geomagnetic storms and other disturbance, with hazards to satellite electronics and humans in space.[1] These energetic electrons are generally believed to be produced by cyclotron resonance with VLF/ELF waves,[2–6] and drift resonance with enhanced ULF waves.[7,8] Particularly, the cyclotron resonant interaction with whistler-mode chorus waves is frequently invoked to be an important acceleration mechanism in the outer radiation belt.[9–15] The bounce-averaged Fokker–Planck equation, treating pitch-angle, energy and cross diffusion, is widely used to describe the wave-particle interaction.[14–17] Several numerical studies have been presented to investigate the wave-particle interaction by solving the Fokker–Planck equation without cross diffusion.[11,14] To overcome the numerical instability due to the large and rapidly varying cross diffusion coefficient, several codes have been developed to fully solve the Fokker–Planck equation including both the diagonal and cross diffusions.[18–20] The previous numerical results suggest that the absence of cross terms can overestimate the energetic electron phase space density (PSD) at given L-shells.[21–23] In this Letter, we investigate the effect of cross diffusion in the resonant interaction between chorus waves and energetic outer zone electrons at different L-shells.

The two-dimensional (2-D) bounce-averaged Fokker–Planck equation for the evolution of phase space density f can be written as[9,17]

$$\frac{\partial f}{\partial t} = \frac{1}{G}\frac{\partial}{\partial \alpha_e}\left[G\left(\langle D_{\alpha\alpha}\rangle\frac{\partial f}{\partial \alpha_e} + \langle D_{\alpha p}\rangle\frac{\partial f}{\partial p}\right)\right] + \frac{1}{G}\frac{\partial}{\partial p}\left[G\left(\langle D_{p\alpha}\rangle\frac{\partial f}{\partial \alpha_e} + \langle D_{pp}\rangle\frac{\partial f}{\partial p}\right)\right], \quad (1)$$

where α_e is the equatorial pitch angle; p is the electron momentum scaled by m_e; $G = p^2T(\alpha_e)\sin\alpha_e\cos\alpha_e$ with $T \approx 1.30\text{–}0.56\sin\alpha_e$; $\langle D_{\alpha\alpha}\rangle$, $\langle D_{pp}\rangle$ and $\langle D_{\alpha p}\rangle = \langle D_{p\alpha}\rangle$ denote bounce-averaged diffusion coefficients in pitch angle, momentum and cross pitch angle-momentum, respectively. For a dipolar geomagnetic field model, those diffusion coefficients are derived to be[24]

$$\langle D_{\alpha\alpha}\rangle = \frac{1}{T}\int_0^{\lambda_m} D_{\alpha\alpha}\frac{\cos\alpha}{\cos^2\alpha_e}\cos^7\lambda d\lambda, \quad (2)$$

$$\langle D_{pp}\rangle = \frac{1}{T}\int_0^{\lambda_m} D_{pp}\frac{(1+3\sin^2\lambda)^{1/2}}{\cos\alpha}\cos\lambda d\lambda, \quad (3)$$

$$\langle D_{\alpha p}\rangle = \frac{1}{T}\int_0^{\lambda_m} D_{\alpha p}\frac{(1+3\sin^2\lambda)^{1/4}}{\cos\alpha_e}\cos^4\lambda d\lambda, \quad (4)$$

where λ is the geomagnetic latitude, λ_m is the maximum latitude where the waves are present; $D_{\alpha\alpha}$, D_{pp} and $D_{\alpha p}$ are local diffusion coefficients given by[25]

$$D_{\alpha\alpha} = \frac{|\Omega_e|^2}{p^2}\left(\frac{p^2}{\gamma^2}I_0 - 2\cos\alpha\frac{cp}{\gamma}I_1 + \cos^2\alpha c^2 I_2\right), \quad (5)$$

$$D_{pp} = c^2|\Omega_e|^2\sin^2\alpha I_2, \quad (6)$$

$$D_{\alpha p} = -c|\Omega_e|^2\sin\alpha\left(\frac{I_1}{\gamma} - \frac{c\cos\alpha}{p}I_2\right), \quad (7)$$

$$I_n = \pi\sum_{\omega_r}\left\{\frac{B_\omega^2}{B_0^2(\lambda)}\left(\frac{\omega_r}{ck_r}\right)^n\left|1-\cos\alpha\frac{p}{\gamma}\frac{dk}{d\omega}\right|^{-1}_{\omega=\omega_r}\right\}, \quad (8)$$

where $n = 0, 1, 2$; γ is Lorentz factor, ω_r (or k_r) is solution of the condition for electrons (with parallel

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velocity $v_\parallel$) in gyroresonance with parallel propagated whistler mode chorus: $\omega - v_\parallel k = |\Omega_e|/\gamma$, B_0 is the ambient magnetic field strength for a dipolar geomagnetic field model:

$$B_0(\lambda) = 3.12 \times 10^4 \frac{\left(1 + 3\sin^2\lambda\right)^{1/2}}{L^3 \cos^6\lambda}\ \text{nT}. \quad (9)$$

To calculate the diffusion coefficients, the dispersion relation and spectrum intensity of chorus waves must be specified. For the standard parallel chorus, the dispersion relation is

$$c^2k^2 = \omega^2 - \frac{\omega\omega_{pe}^2}{\omega - |\Omega_e|}, \quad (10)$$

where $|\Omega_e|$ and ω_{pe} denote the local electron gyrofrequency and plasma frequency respectively; c is the speed of light; ω is the wave frequency, k is the wave number. The whistler mode chorus is distributed over a Gaussian frequency band peaked at ω_m with half width $\delta\omega$, a lower cutoff ω_l, an upper cutoff ω_u:

$$B_\omega^2 = \begin{cases} B_n B_t^2 \exp\left[-\frac{(\omega-\omega_m)^2}{\delta\omega^2}\right] & \text{for } \omega_l \le \omega \le \omega_u, \\ 0 & \text{otherwise}, \end{cases} \quad (11)$$

where B_t represents the total magnetic strength and the parameter B_n is determined by

$$B_n = \frac{2}{\pi^{1/2}\delta\omega}\left[\text{erf}\left(\frac{\omega_u - \omega_m}{\delta\omega}\right) + \text{erf}\left(\frac{\omega_m - \omega_l}{\delta\omega}\right)\right]^{-1}. \quad (12)$$

Table 1. The spectra properties for equatorial chorus wave.

	$\omega_l(\lvert\Omega_e\rvert)$	$\omega_u(\lvert\Omega_e\rvert)$	$\delta\omega(\lvert\Omega_e\rvert)$	$\omega_m(\lvert\Omega_e\rvert)$	B_t[pT]	λ_m
Dayside	0.1	0.3	0.1	0.2	$10^{0.75+0.04\lambda}$	35°
Nightside	0.05	0.65	0.15	0.35	50	15°

The chorus waves are often observed in the magnetic local time (2200-1300MLT),[26] and have different spectrum properties in dayside and nightside of the Earth. Based on the previous studies,[14,27,28] the parameters we used in this model are given in Table 1. The weighted diffusion coefficients can be obtained by applying 25% drift averaging for both dayside and nightside to describe the wave characteristic around the earth.

Following the previous models,[9,22] the initial distribution of electron is taken as $f|_{t=0} = \exp[-(E - 0.2)/0.1]\sin\alpha_e/p^2$ in this study, with kinetic energy E. The boundary conditions are[29] $f(0.2\,\text{MeV}) = f_{t=0}(0.2\,\text{MeV})$ at lower boundary, $f(5.0\,\text{MeV}) = 0$ at upper boundary; $f = 0$ at the loss-cone $\alpha_e = \alpha_L$, $\partial f/\partial\alpha_e = 0$ at $\alpha_e = 90°$. We adopt the hybrid finite difference (HFD) code[17,23,30–33] to fully solve the Fokker–Planck equation. The time step set as 5 s to avoid the numerical instability.

Figure 1 plots the weighted diffusion coefficients in pitch angle, momentum and cross pitch-angle-momentum for 0.5 MeV and 1.0 MeV electrons at $L =$ 3.5, 4.5, 5.5 and 6.5. It is found that all the diffusion rates have a significant linear enhancement with L increasing in the pitch angles range of 0°–50° and around 90°. Specifically, diffusion rates at $L = 6.5$ are found to be 5–10 times larger than that for $L = 3.5$ at pitch angles around 25° and 90°. However, the cross diffusion rate for 1.0 MeV electrons around pitch angle 70° decreases with L increasing.

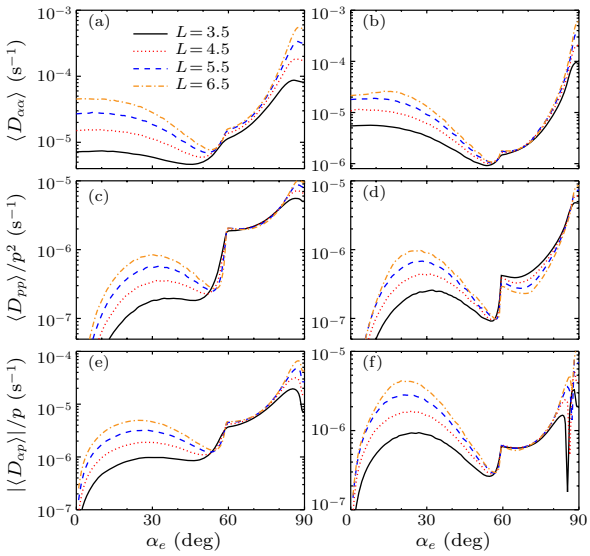


Fig. 1. Weighted bounce-averaged diffusion coefficients of pitch angle, momentum, and cross in units of (s^{-1}) for whistler mode chorus for 0.5 MeV (left) and 1.0 MeV (right) electrons at different indicated L-shells.

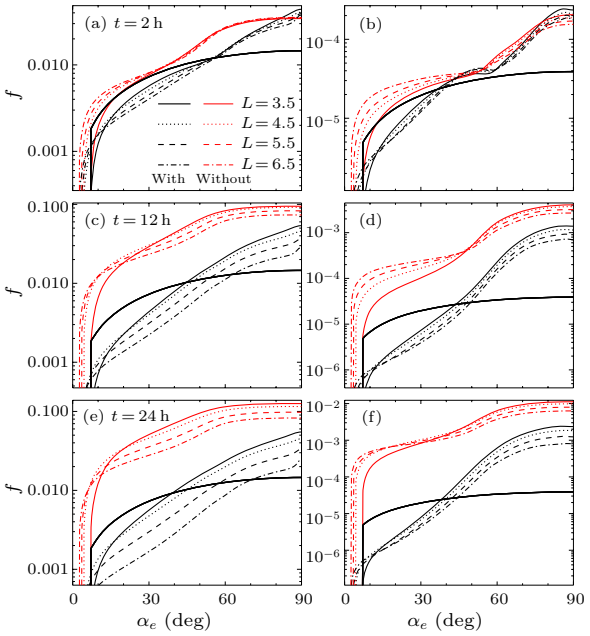


Fig. 2. Electrons PSD as a function of pitch angle for 0.5 MeV (left) and 1.0 MeV (right) after specified times $t = 2$, 12, 24 h at different indicated L-shells. The bold black lines represent the initial value of PSD. The red lines and slender black lines denote the PSD evolution with and without cross diffusion terms respectively.

The PSD evolution of electrons with energies

0.5 MeV and 1.0 MeV at different L-shells are shown in Fig. 2. The enhancement of PSDs for 0.5 MeV electrons in 24 h are found to be generally less than one order of magnitude in the simulations with and without cross diffusion. In the absence of cross diffusion, PSD for 1.0 MeV electrons at $L = 3.5$, 4.5, 5.5 and 6.5 after 24 h is found to increase by about 10, 50, 60 and 70 times at pitch angles $< 35°$, and increases by about 275, 250, 200 and 150 times at pitch angles 35°–90°, respectively. In the presence of cross diffusion, PSD for 1.0 MeV electrons after 24 h decreases by about 25, 12, 10 and 8 times at $L = 3.5$, 4.5, 5.5 and 6.5 at small pitch angles, and it increases by about 55, 45, 30 and 20 times at larger pitch angles, respectively. These results conform the previous findings that the resonant interaction with whistler-mode chorus presents an efficient acceleration mechanism for outer radiation belt electrons,[10,12–14] and neglecting the cross diffusion can lead to an overestimate of PSDs especially at the lower pitch-angles.[22,23,29]

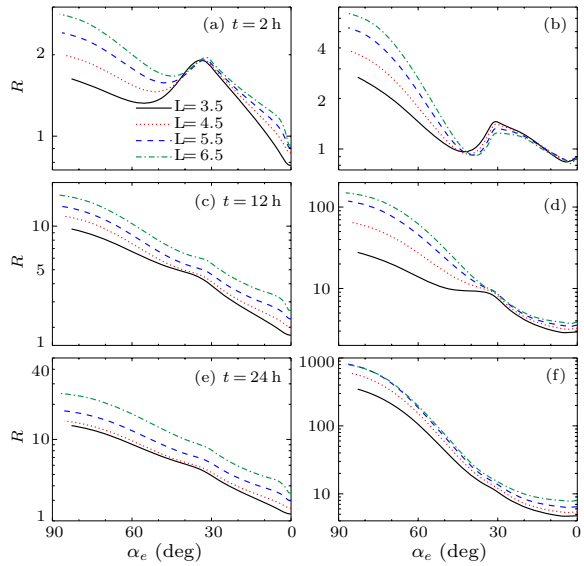


Fig. 3. The ratio of electrons PSD without cross terms to with cross term for 0.5 MeV (left) and 1.0 MeV (right) after specified times $t = 2$, 12, 24 h at different indicated L-shells.

The evolution of ratio R between electron PSDs from simulations without and with cross diffusion terms at each L-shell is shown in Fig. 3. It is clear seen that the ratio R has values about 1.0 for both 0.5 and 1.0 MeV electrons at $t = 2.0$ h, indicating that the influence of omitting the cross terms is unconspicuous during the initial times. The R values are found to increase with time increasing. After 24 h, the R values for 0.5 MeV electrons at $L = 3.5$, 4.5, 5.5 and 6.5 are about 12, 14, 18 and 24 near the loss cone, respectively. The ratios R decrease with pitch angle increasing, and they become 2.2, 2.5, 3 and 3.4 at pitch angle 90°, respectively. The corresponding ratios for 1.0 MeV electrons are about 350, 600, 800 and 800 near the loss cone, and 5, 5.5, 6.5 and 8 at $\alpha_e = 90°$, respectively. Obviously, the ratios R tend to increase with L increasing, suggesting that the cross diffusion plays a more important role at larger L-shells.

In summary, we have investigated the resonant interaction between energetic outer radiation belt electrons and whistler-mode chorus at different L-shells based on the 2-D bounce-averaged Fokker–Planck equation. The diffusion rates for energetic electrons are found to have an obvious enhancement with L increasing at pitch angles range of 0°–50° and around 90°. Specifically, diffusion rates for $L = 6.5$ are about 5–10 times larger than that for $L = 3.5$ at these pitch angles. The energetic electron PSD evolution at different L-shells has been simulated using the HFD code. After 24 h, ∼MeV electron PSD from simulation without cross diffusion at $L = 3.5$, 4.5, 5.5 and 6.5 increases by about 10, 50, 60 and 70 times at pitch angles $< 35°$, and about 275, 250, 200 and 150 times at pitch angles 35°–90°, respectively. ∼MeV electron PSD from simulation with cross diffusion at $L = 3.5$, 4.5, 5.5 and 6.5 decreases by about 25, 12, 10 and 8 times at small pitch angles, and increases by about 55, 45, 30 and 20 times at larger pitch angles, respectively. These PSD variation scales are comparable with the previous observations.[34] The ratios between ∼MeV electron PSDs from simulations without and with cross diffusion at $L = 3.5$, 4.5, 5.5 and 6.5 are about 350, 600, 800 and 800 near the loss cone, and decrease to about 5, 5.5, 6.5 and 8 at pitch angle 90°, respectively. These results demonstrate that the neglect of cross diffusion can cause the overestimate of energetic electron PSDs, and the cross diffusion plays a more significant role at larger L-shells. It should be noted that current numerical results depend on the wave model, initial and boundary conditions. Observation data-driven modeling works need to be done in the future.

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