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Neutron Star Motion in the Disk Galaxy *

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The neutron star motions are based on the undisturbed finitely thick galactic disk gravitational potential model. Two initial conditions, i.e. the locations and velocities, are considered. The Monte Carlo method is employed to separate rich diversities of the orbits of neutron stars into several sorts. The Poincaré section has the potential to play an important role in the diagnosis of the neutron star motion. It has been observed that the increasing ratio of the motion range vertical to the galactic plane to that parallel to the galactic plane results in the irregularity of neutron star motion.

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The galactic gravitation field is considered as a power and governs the neutron stars and other dynamical objects.[1] The Galactic gravitational potential is based on three components, i.e., disk, sphere and halo, which may be well fitting for the Galactic rotation curve.[1] Neutron stars would be particularly interesting, because many such sources are dynamically for a long duration, allowing the follow-up investigation of increasing precision,[3] and because it is believed that the characteristics of these motions will provide us more knowledge of these exotic stars.[4] In Paczyński's Galactic gravitational potential,[2] different from the spherical symmetry of the gravitational potentials of Galactic sphere and halo components, the form of gravitational potential of Galactic disk looks comparatively peculiar with the only rotational symmetry in the z direction and mirror symmetry to the Galactic plane,[5] which is a semi-empirical gravitational potential of the disk galaxy. Peng *et al.* derived a vigorous three-dimensional (3-D) solution of finitely thick disk galaxy; furthermore, they studied the dynamics of the galactic disk with finite thickness;[6−10] methods were also put forward for determining the thickness of spiral galaxies and mass-to-light ratios of nearly face-on spiral galaxies..[11,12]

In addition, the distribution of neutron stars in the undisturbed disk galaxy was simulated.[13,14] However, in this Letter we aim at the orbits of single neutron stars in the disk galaxy, where the initial conditions of the stars, i.e. initial 3-D velocities and locations, are chosen with the Monte Carlo method,[15] then their orbits are numerically integrated. We would utilize the method of the Poincaré section to explore their motions. On the numerical study of the kinematics and dynamics of Galaxy, the 3-D N-body model is realized by self-consistent evolution.[16] For example, Hobbes *et al.* found the mean 3-D velocity of pulsars to be $400 \pm 40\,\mathrm{kms}^{-1}$, while a study by Lyne *et al.* showed that the mean 3-D speed to be $130 \pm 30\,\mathrm{kms}^{-1}$. Despite these different population syntheses, the two populations are consistent with the idea that all neutron stars share the same velocity distribution.[15,17]

The density distribution of an undisturbed finite disk galaxy is supposed to be the exponential profiles in both parallel and vertical directions to the galactic disk:[8,13,14]

$$\rho(r,z) = \rho(r,0)e^{-\alpha|z|} = \frac{\alpha}{2}\sigma(r)e^{-\alpha|z|}, \tag{1}$$

where $\alpha = \frac{1}{h_z}$ with h_z being the scale height of the galactic disk; $\sigma(r) = \sigma_0 e^{-\frac{r}{h_d}}$ with h_d being the scale length of the galactic disk.

The Poisson equation of gravitational potential with the above density distribution is

$$\nabla^2\phi(r,z) = -2\pi G\alpha\sigma(r)e^{-\alpha|z|}, \tag{2}$$

where G is the gravitational constant. The solution to the above equation is

$$\begin{aligned}
\phi(r,z) &= -\pi G\alpha \int_0^\infty J_0(\beta r)S(\beta)\Theta(\alpha,\beta;z)d\beta,\\
S(\beta) &= \int_0^\infty rJ_0(\beta r)\sigma(r)dr.\\
\Theta(\alpha,\beta;z) &= \int_{-\infty}^{\infty} e^{-\beta|z-z'|-\alpha|z'|}dz'\\
&= \frac{2}{\beta^2-\alpha^2}(\beta e^{-\alpha|z|} - \alpha e^{-\beta|z|}),
\end{aligned} \tag{3}$$

where $J_0(\beta r)$ is the 0-rank Bessel function. In the calculation we adopt the parameters of disk galaxy as those of the Galactic disk. They are $h_d = 2.5\,\mathrm{kpc}$, $\sigma(R_0) = 54\ M_\odot/\mathrm{pc}^2$, $R_0 = 8.5\,\mathrm{kpc}$ is equal to the distance between the Sun and the Galactic center,[8] $h_z = 0.325\,\mathrm{kpc}$.

The kinematic differential equation of the stars reads

$$\ddot{\boldsymbol{R}} = -\nabla\phi(\sqrt{x^2+y^2}, z),\ |\boldsymbol{R}| = \sqrt{x^2+y^2+z^2}, \tag{4}$$

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where the orbits of the star were numerically integrated with a fifth-order Runge–Kutta method.[18] We calculated a lot of orbits of stars, observed the configurations of these orbits and then made a category to them. It was utilized the method of section to study the motion of objects in galaxies.[19] We here also use this method of Poincaré section to study the complex motion of stars.

Supposing that the dimension of phase space of object motion is n, we can select the other $(n-1)$-D hypersurface section while keeping one coordinate constant in the n-D phase space to study the motion: when the object passes through the section it leaves a point in the section, the map of the discrete points in the section reflects the object's continual motion in the phase space. Using section to study dynamic system was first introduced by Poincaré.[19,20]

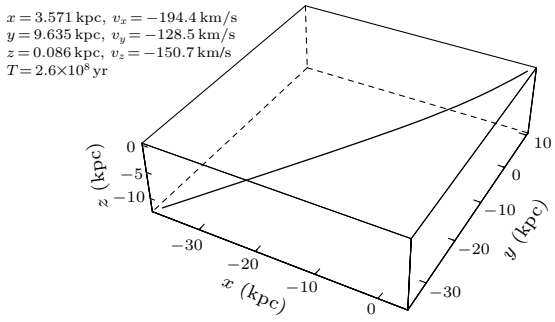


Fig. 1. The 3-D trajectory of an escaped neutron star is like conic hyperbola. The neutron star's initial condition is under the figure. T indicates the time to which the orbit is calculated.

The stars originally have 6 dimensions (x, y, z, v_x, v_y, v_z) in the phase space, but their total energies E_t and angular momentums J_z in the z direction are conservative, which can be used to remove two variables (v_y, v_z). Thus we can describe their motions in 4-D (x, y, z, v_x) phase space and use the 3-D section. Calculating the 3-D sections needs very high accuracy, we must keep more significance digits, so this kind of calculating spent a rather long time. In fact we can only calculate about $20 \sim 30$ intersection points between the star orbits and their 3-D sections per day. In comparison, Paczyński[2] used the total energy E_t to control the accuracy of his calculation and kept it better than 1 part in 10^6, the maximum age of neutron stars to which he calculated is $10^{10}a$. However owing to the angular momentum J_z being conservative, for a calculated star we compared its initial J_z with that of the 803rd intersection point in its Poincaré section which is about $\sim 10^{11}a$, and found the relative error between the two is surprisingly less than 2 parts in 10^{11}! Certainly the accuracies change in the different orbits, and generally the simpler orbits have the accuracies higher than the more complex ones. In addition, since this study is idealized, we suppose that there is no evolution in the galactic gravitational potential and we ignore the interstellar friction.

The Monte Carlo method effectively covers various possible cases of the initial conditions of neutron stars, which is the reason of our using this method to select the initial conditions of neutron stars. By inspecting a large amount of orbits of stars under the different initial conditions, we find that although the motion ranges of neutron stars are different, their motion configurations are approximately attributed to 4 kinds of orbits. The first kind of orbit is shown in Fig. 1, which represents the orbits of escaped stars and is like conic hyperbola. T in the figure indicates the time to which we calculate the orbit and the initial condition is also put under the figure.

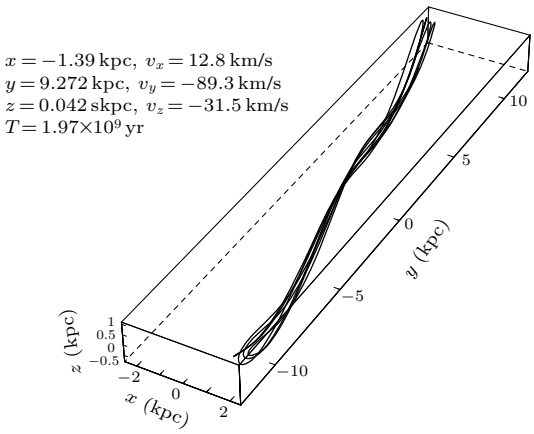


Fig. 2. The neutron star's initial $\frac{x}{y} \approx \frac{v_x}{v_y}$, and its trajectory degenerates into a straight line through the galactic center in the projection onto the galactic plane with oscillation in z.

The second kind of orbit is similar to Fig. 2, where their initial conditions are approximately consistent with $\frac{x}{y} \approx \frac{v_x}{v_y}$, and their projective orbits onto the x-y plane degenerate into approximately straight line, meanwhile they make oscillations along the z direction. Certainly these cases are comparatively seldom.

The last two kinds of orbits of star are all those circulating around the galactic center. The difference between them is that the eccentricities of the projections of the third kind of trajectories of stars on the galactic plane are small, which are about circles as Fig. 3(b) shows, whereas the eccentricities of the projections of the fourth kind of trajectories of stars on the galactic plane are not small, which do precession around the galactic center like ellipses as Fig. 4(b) shows.

For the two kinds of orbits we utilize the (x, z, v_x) Poincaré hypersurface section of the phase space at $x > 0$, $y = 0$ to study their 3-D motions. T_P in these two figures marks the time till which we calculate the Poincaré sections. Besides these the projections of the 3-D Poincaré hypersurface sections onto x–z and x–v_x planes are also plotted respectively in (d) and (e) of Figs. 3 and 4 for further inspection.

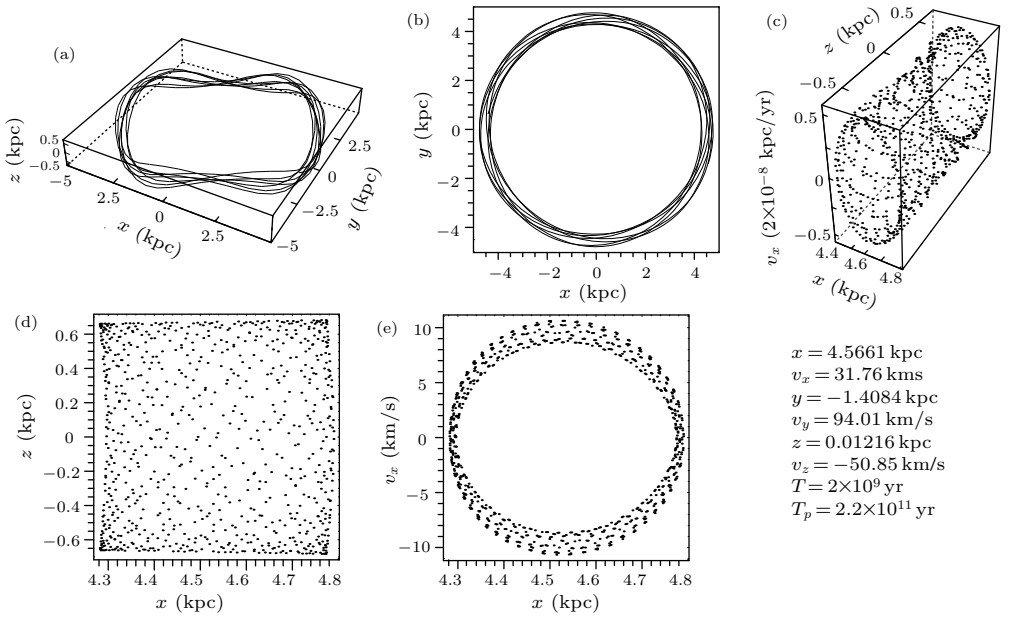


Fig. 3. The eccentricity of the projection of the neutron star's 3-D trajectory on the galactic plane is small. (a) The 3-D trajectory; (b) the projection on the galactic plane of the 3-D trajectory; (c) the 3-D Poincaré section $x > 0$, $y = 0$ of the 3-D trajectory; (d) the projection on x-z plane of the 3-D Poincaré section; (e) the projection on x-v_x plane of the 3-D Poincaré section.

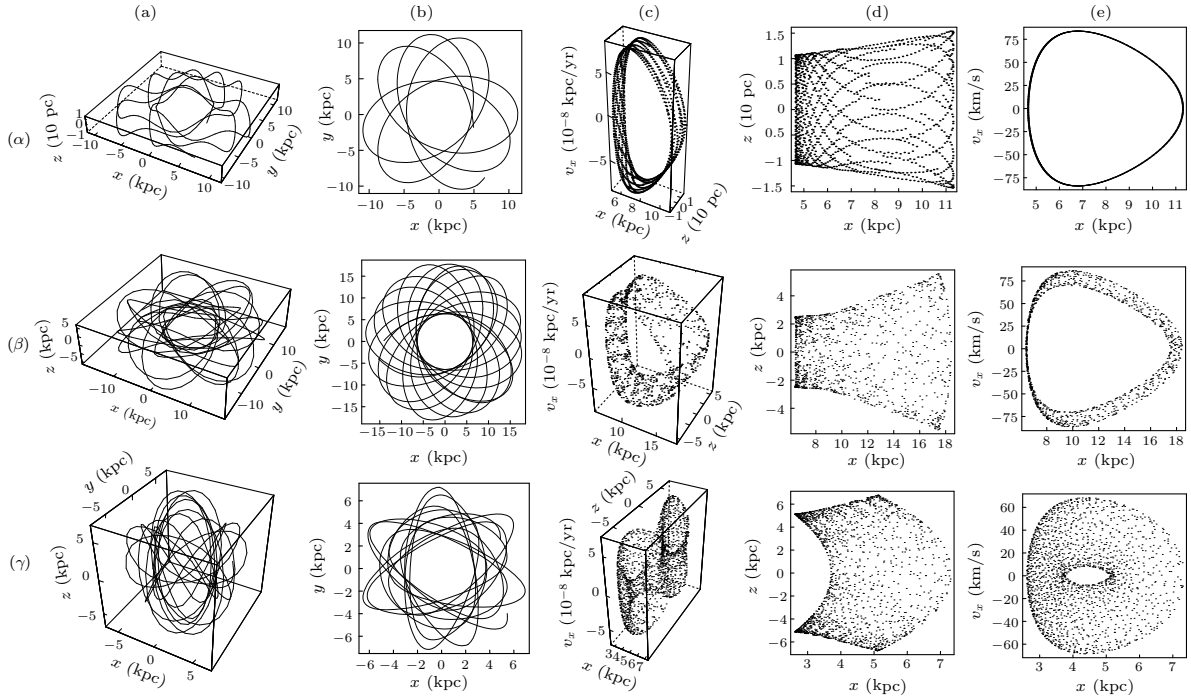


Fig. 4. $(\alpha) - (\gamma)$ The eccentricities of the projections of the neutron stars' 3-D trajectories on the galactic plane are not small, which do precessions around the galactic center. (a) The 3-D trajectory; (b) the projection on the galactic plane of the 3-D trajectory; (c) the 3-D Poincaré section at $x > 0$, $y = 0$ of the 3-D trajectory; (d) the projection on x-z plane of the 3-D Poincaré section; (e) the projection on x–v_x plane of the 3-D Poincaré section.

More special attention should be paid to the fourth kind of orbits in Fig. 4, since they are nontrivial. From Figs. 4(α)–4(γ) the stars' motions turn more irregular. From (c) and (d) of Fig. 4(α) we see that the intersection points distribute in some regular lines on the Poincaré hypersurface section, especially in (e) of Fig. 4(α) the projection of the Poincaré section on x–v_x plane is essentially an enclosed curve, which means that the motion is an quasiperiodic orbit (in fact if the motion is vigorously periodic, we could expect that after some time the star should return back to the same intersection point on the hypersurface section). In addition, From (a) of Fig. 4(α) we see the motion of Fig. 4(α) basically can be seen as the independent superposition of the rotation in the Galactic plane and the z oscillation vertical to the Galactic plane. However, in (c) and (d) of Fig. 4(β) we cannot see the regular lines as in Fig. 4(α), but there seems to exist some local collections like (c) and (d) of Fig. 3. Moreover in (e) of Fig. 4(β) the front enclosed curve in (e) of Fig. 4(α) is some dispersed and broadened, but with the left part still in a segment of line. All these show that the star's motion in Fig. 4(β) shows some certain irregularity. Finally we see that the segment of line in the left part of (e) of Fig. 4(β) disappears in (e) of Fig. 4(γ) by being scattered and the intersection points distribute more evenly on the Poincaré hypersurface section, which represents a comparatively obviously irregular motion.

In order to explore the reason of transforming from Figs. 4(α)–4(γ) we studied a large number of the orbits of the fourth kind, and finally we are aware that this transforming has nothing to do with the absolute motion ranges of stars, which might be called the configuration analogy of neutron star motion in different ranges of the Galaxy under Peng's gravitational potential, whereas is decided by the relative motion ranges, i.e., the ratios of motion ranges in the z direction to motion ranges in $r = \sqrt{x^2 + y^2}$ direction. With the increase of these ratios, the stars' motions turn more irregular. In fact, we could see that the inclination angle of the orbital plane to the z-axis in Fig. 4(γ)(a) changes very fiercely.

Thus the above are the four categories related to the orbits of neutron star in the disk galaxy under different initial conditions under the undisturbed finitely thick disk gravitational potential and these categories are founded on our observation of that the configurations of neutron stars moving in different galactic ranges have a certain analogy in both the true x-y-z space and the Poincaré section.

Finally, we should briefly discuss the neutron stars' motion in the Paczyński's Galactic gravitational potential. Different from Peng's gravitational potential, Paczyński's gravitational potential has the components from the sphere and halo, so the neutron stars will have obviously different motions from that in Peng's gravitational potential. In different locations of the Galaxy, the neutron stars' motion will be mainly affected by different components of Paczyński's gravitational potential, certainly there also exist some ranges in which different Paczyński's gravitational potential's components have the approximately equal effect. For example, if the neutron star is in the centric ~ 1 kpc region of the Galaxy, due to no gravitational force of the spherically symmetric hollow sphere material on the inner of its own, the neutron star will feel little effect from the sphere and halo. However, when the neutron star moves out, it will feel the increasing effect of the sphere and halo which might be very quickly comparable to the effect of the disk; finally in the far region from the Galactic center, the neutron star mainly feel the effect of the spherically symmetric halo component, and the effect of the disk and sphere is in a position of subordination. As is well known, the reason why the Galactic circular curve does not decrease very quickly in the large distance from the Galactic center according to the effect of the Galactic sphere and disk is just owing to the existing of the effect of the Galactic halo. Certainly, the above analysis is only qualitative, the precise answer to the neutron stars' motion in Paczyński's gravitational potential needs the accurate numerical calculation.

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