Effect of the Tensor Force on Charge-Exchange Spin-Dependent Multipole Excitations *

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The charge-exchange spin-dipole (SD) and spin-quadrupole (SQ) strength functions of ^{90}Zr are calculated with and without the tensor terms of the Skyrme interaction in self-consistent HF+RPA approach. It is found that, in SD and SQ transitions, the RPA correlations associated with the tensor terms shift dramatically the strengths of $(Y_l\sigma)_{\lambda=l-1}$ and $(Y_l\sigma)_{\lambda=l}$ modes upward and downward, respectively, and also shift the strengths of $(Y_l\sigma)_{\lambda=l+1}$ modes upward. The coupling between $(Y_{l=\lambda-1}\sigma)_\lambda$ and $(Y_{l=\lambda+1}\sigma)_\lambda$ modes arising from the tensor correlation is noticeable. The RPA tensor correlations produce strengths of SD and SQ modes, which are distributed in a much wider energy range, and the $(Y_l\sigma)_{\lambda=l-1}$ modes dominate the high energy part of the strength functions. These energy shifts and coupling effects of different modes can be understood qualitatively by expressing a finite range tensor force in a separable form.

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Studying nuclei which lie away from the β-stability line is producing new progression in our understanding of nuclear structure. In particular, the evolution of single-particle energies along isotopic or isotonic chains has elucidated the specific effect of the tensor force,[1−3] which has been almost forgotten, or at least overlooked, for many years. In our previous papers,[4,5] the Skyrme's tensor terms[6,7] are included in the self-consistent HF+RPA model to study the Gamow–Teller (GT) and charge-exchange 1^+ Spin-Quadrupole (SQ) transitions in ^{90}Zr and ^{208}Pb. It was shown that the main GT peak is moved downward by about 2 MeV, while the coupling between GT and 1^+ SQ modes induced by the RPA tensor correlations shifts about 10% of the GT strength to the energy region higher than 30 MeV, which is supposed to be the energy region of $2p-2h$ configurations.[9] In Ref. [10], the tensor coupling effects on magnetic moments of hypernuclei are studied.

The spin-dependent charge-exchange operators are given as

$$\hat{O}^\lambda_\pm = \sum_i t^i_\pm r^l_i (Y^i_l \sigma^i)^\lambda, \tag{1}$$

where $t_\pm = \frac{1}{2}(t_x \pm i t_y)$ are the isospin operators, and $l = 1, 2, 3$ and 4 correspond to spin-dipole (SD), SQ, spin-octopole (SO), and spin-hexadecapole (SH) modes, respectively. In this Letter, the effects of the tensor force on charge-exchange SD, SQ and some relevant SO and SH excitations are studied, especially for the multipole dependence of the tensor effect on the strength distributions.

In the HF+RPA calculations, the same tensor terms already used in Ref. [4] are employed, that is, the Skyrme SIII parameter set has been supplemented with the tensor contribution given in Ref. [2], in which $T = 1008\,\text{MeV}\cdot\text{fm}^5$ and $U = -432\,\text{MeV}\cdot\text{fm}^5$. Some of the single-particle energies of proton and neutron in the HF potential with and without the tensor terms in ^{90}Zr are listed in Table 1. The spin-orbit splitting of proton is decreased, while that of the neutron is increased, which can be understood by the mechanism stated in Ref. [3].

Table 1. Single-particle energies of proton and neutron in the HF potential with and without the tensor terms in ^{90}Zr.

	Proton (MeV)		Neutron (MeV)	
State	No tensor	With tensor	No tensor	With tensor
$1f_{7/2}$	−16.2	−15.1	−22.0	−23.3
$1f_{5/2}$	−10.6	−11.9	−17.5	−15.5
$2p_{3/2}$	−8.7	−8.4	−16.4	−16.5
$2p_{1/2}$	−6.9	−7.1	−14.7	−14.2
$1g_{9/2}$	−6.2	−5.0	−11.8	−13.5
$1g_{7/2}$	1.4	0.1	−5.2	−3
$3s_{1/2}$	3.5	3.7	−4.6	−4.0
$2d_{5/2}$	1.7	2.1	−6.0	−6.3
$2d_{3/2}$	4.1	3.8	−3.7	−3.2
$1h_{11/2}$	4.1	5.3	−1.64	−3.4
$1h_{9/2}$	12.3	11.6	6.1	7.1

In the present RPA calculation, the harmonic oscillator basis is used. We include all states up to a maximum value of the principal quantum number $N_{\max}$, equal to 10 and 20 for SD and SQ calculations, respectively. The discrete RPA peaks have been smoothed by using a Lorentzian averaging with a width of 1 MeV. We have performed three kinds of

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calculations: first, the tensor terms are not included; secondly, the tensor terms are included only in HF, with the aim of displaying the effects of the changes in single-particle energies caused by the tensor term on the strength distribution; finally, the tensor terms are included in both HF and RPA. The results of these calculations are shown in figures by '00', '10' and '11', respectively.

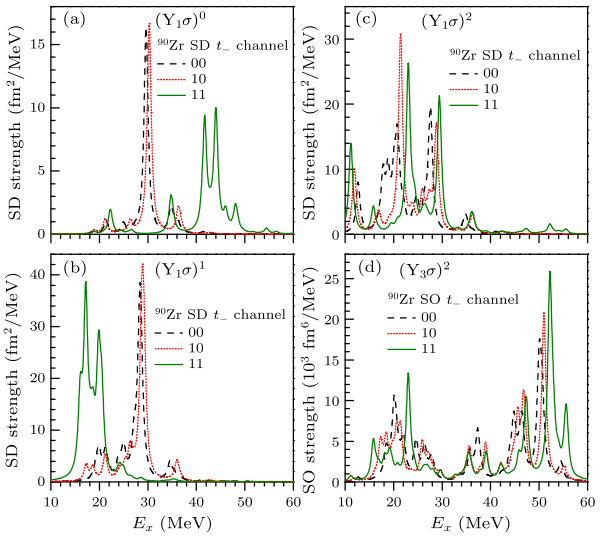


Fig. 1. (Color online) The charge-exchange SD$_-$ and 2$^-$ SO$_-$ strength distributions in ^{90}Zr. See the text for details.

Table 2. The SD NEWSR S_- distribution for ^{90}Zr in different excitation energy regions. The analytical NEWSR is calculated by using the formula in Ref. [8]. See the text for detailed discussion of the effects of the tensor terms.

Mode	Type of calculation	S_- 0–38 MeV	S_- 38–60 MeV	NEWSR RPA	NEWSR analytic
	00	33.3	1.13	16.45	16.4
0^-	11	10	36	16.1	16
	00	117.2	2.6	49.3	49.2
1^-	11	157.1	1.4	48.4	48.2
	00	126.3	2.2	81.8	82
2^-	11	124.3	9.5	80	80.3

The strength distributions of 0$^-$, 1$^-$ and 2$^-$ SD$_-$ and 2$^-$ SO$_-$ transitions of ^{90}Zr are displayed in Fig. 1. As shown in Figs. 1(a) and 1(b), the centroid of the 0$^-$ strength is shifted upward by about 16 MeV, while the centroid of the 1$^-$ strength is shifted downward by about 8 MeV due to the RPA tensor correlation. The effect of single-particle energy changes from the tensor contribution is very weak compared to the RPA residual interaction. The strength distributions of 2$^-$ SD$_-$ and SO$_-$ transitions are displayed in Figs. 1(c) and 1(d) respectively, which are presented in such a way that the coupling effects of the tensor terms are clear. The centroid of 2$^-$ strength is shifted upward only about 2 MeV. The non-energy weighted sum rule (NEWSR) strengths are given in Table 2 for SD transitions.

The analytic formula NEWSR provides the information of accuracy of the present numerical calculations in comparison between the values listed in fifth and sixth rows in Table 2. However, the strength of both t_- and t_+ channels are increased with the inclusion of the tensor correlation. For the 0$^-$ mode, more than 95% of the strength is found below $E_x = 38$ MeV without the tensor correlation, while nearly 80% of the strength is shifted to the higher energy region of $E_x > 38$ MeV when the tensor correlation is included. This leads to the result that about 77% of the strength in this region is 0$^-$ strength.

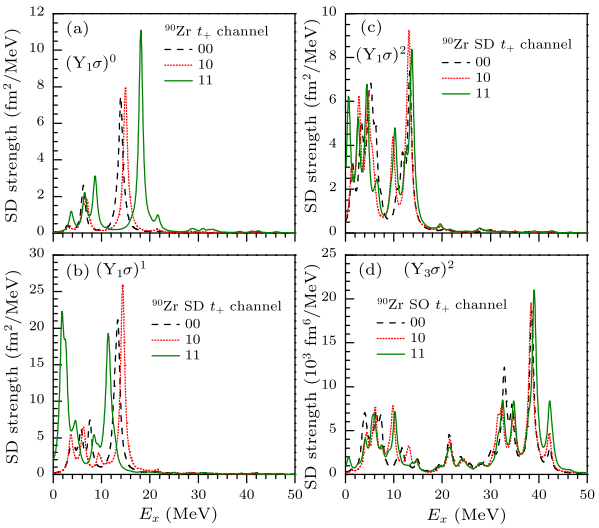


Fig. 2. (Color online) The charge-exchange SD$_+$ and 2$^-$ SO$_+$ strength distributions in ^{90}Zr. See the text for details.

The strength distributions of 0$^-$, 1$^-$ and 2$^-$ SD$_+$ and 2$^-$ SO$_+$ transitions of ^{90}Zr are displayed in Fig. 2. We see that the qualitative features induced by RPA tensor correlation are the same as those in the SD$_-$ case, but the effects are quantitatively much weaker.

The total strength distributions of the SD transitions in the t_- and t_+ channels are compared with experimental results in Figs. 3(a) and 3(b), respectively. After the tensor terms are included, the total NEWSRs for t_- and t_+ channels are increased by about 13% and 18%, respectively. The total SD$_-$ strengths are distributed in a much wider energy range by the RPA tensor correlations. The centroid energies are shifted downward by about 1.4 MeV, for both the channels. With the present chosen parameters, the inclusion of the tensor terms can not well reproduce the low-energy part of the experimental data.[8] Since the RPA calculation without the tensor terms gives the total main peak higher than the experimental data and the tensor terms can shift the main peak downward, the tensor terms shed light on well reproduce of the experimental data. As the presently used tensor interactions are added perturbatively to the existing parameter set, the tensor parameters and the previous Skyrme parameters should be refitted if we want to reproduce the experimental data of the SD states. Furthermore, especially in the t_- channel the exper-

imental data show that there is high-energy strength which can be accounted for with the inclusion of the tensor terms. Since this high-energy t_- strength lying above 40 MeV has almost pure 0^- character, we claim that disentangling the different components of the SD$_-$ strength (studying the analyzing powers by possibly polarized projectile experiments[11]) can provide useful information to assess the magnitude and strength of the tensor force, as well as the spin-isospin component of the Skyrme interaction which is still not constrained so well.

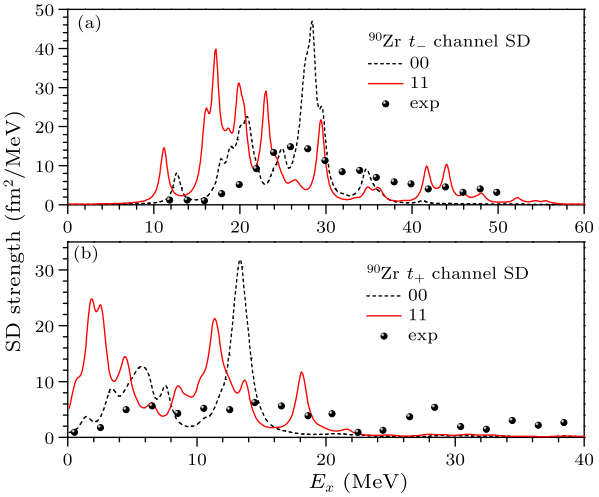


Fig. 3. (Color online) The total charge-exchange SD± strength distributions in ^{90}Zr. The t_- and t_+ channels are displayed (a) and (b), respectively. The experimental results from Ref. [8] are labelled by "exp". The discrete RPA peaks have been smoothed by using a Lorentzian averaging with a width of 1 MeV. See the text for details.

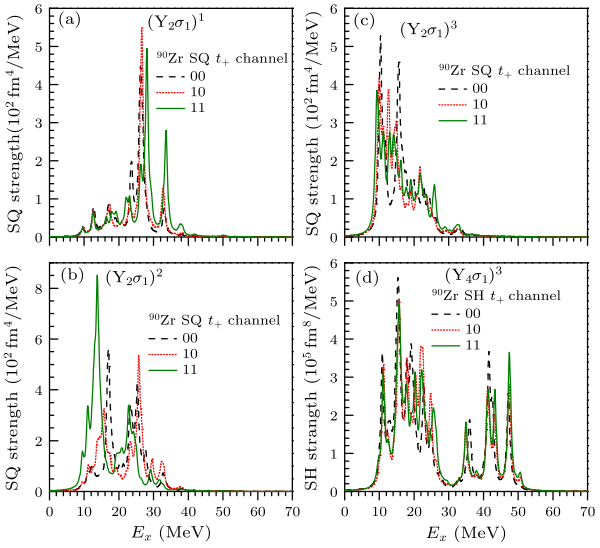


Fig. 4. (Color online) The charge-exchange SQ$_-$ and 3^- SH$_-$ strength distributions in ^{90}Zr. See the text for details.

The strength distributions associated with the 1^+, 2^+, 3^+ SQ$_-$ transitions and with the 3^+ SH$_-$ transitions of ^{90}Zr are displayed in Fig. 4 As shown in panel (a), the centroid of the 1^+ SQ strength is shifted upward by 11 MeV whereas in panel (b) one can notice that the centroid of the 2^+ SQ strength is shifted downward by 7.4 MeV by the RPA tensor correlations. The strength distributions associated with the 3^+ SQ$_-$ and SH$_-$ transitions are displayed in panels (c) and (d), respectively. They are presented in the same column in order to show the coupling effects of the tensor terms more clearly. The centroid of the 3^+ SQ$_-$ strength is shifted upward by about 3 MeV while the centroid of the total SQ$_-$ strength is nearly unchanged.

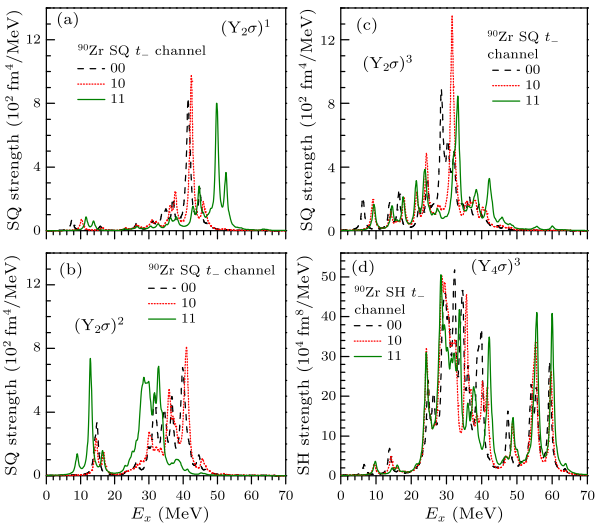


Fig. 5. (Color online) The charge-exchange SQ$_+$ and 3^- SH$_+$ strength distributions in ^{90}Zr. See the text for details.

The strength distributions of the 1^+, 2^+ and 3^+ SQ$_+$ transitions, as well as of the 3^+ SH$_+$ transitions of ^{90}Zr are displayed in Fig. 5. It can be seen that the qualitative features induced by the RPA tensor correlations are the same as those in the SQ$_-$ case.

We also perform the same calculations in ^{90}Zr by employing the Skyrme parameter set SLy5, plus the tensor force used in Ref. [3], the obtained results are similar to the above ones. It seems that if one wants to reproduce the experiment peak of SD data by using Skyrme force with tensor terms, the parameters including the tensor terms need to be refitted.

In order to qualitatively understand our calculated results, we start from the finite range tensor force $F(|\boldsymbol{r}|)S_{12}$, in which the form factor $F(|\boldsymbol{r}|) = V_T^{(e)} + V_T^{(o)}$ is positive, and

$$S_{12} = (\hat{\boldsymbol{r}} \cdot \sigma_1)(\hat{\boldsymbol{r}} \cdot \sigma_2) - \frac{1}{3}(\sigma_1 \cdot \sigma_2). \qquad (2)$$

The relations of the triplet-even and triplet-odd tensor forces $V_T^{(e)}S_{12}$ and $V_T^{(o)}S_{12}$ to those of Skyrme's zero range tensor force were studied in a short range limit in Ref. [6], in which $V_T^{(e)}$ and $V_T^{(o)}$ correspond to positive T and U, respectively. Presently used tensor parameters in our calculations are positive T and neg-

ative U. In Refs. [2,3], the parameters T and U are chosen in such a way to improve the absolute values and the isotopic (or isotonic) trends of single-particle energies. This criterion limits the possible choice on sign of T and U.

By this choice of the parameters, the zero range tensor force in our calculations corresponds to $(V_T^{(e)} - V_T^{(o)})S_{12}$. By anti-symmetrizing them, the isospin part of the tensor force can be written as

$$V_T = -\frac{1}{2}F(|\boldsymbol{r}|)S_{12}\tau_1 \cdot \tau_2. \tag{3}$$

In general, $F(|\boldsymbol{r}|)$ can be expressed by means of a partial wave expansion as

$$F(|\boldsymbol{r}|) = \sum_{lm} f(l, r_1, r_2)Y^*_{lm}(1)Y_{lm}(2), \tag{4}$$

and for finite range force, it can be expected that the $l = 0$ is the dominant term and the next is $l = 1$, and so on.

For the case $l = 0$,

$$\begin{aligned} V_T^{l=0} = & -\sqrt{\frac{1}{6}}f(l=0,r_1,r_2)\{[(Y_0(1)\sigma_1)_1(Y_2(2)\sigma_2)_1]_0 \\ & + [(Y_2(1)\sigma_1)_1(Y_0(2)\sigma_2)_1]_0\}\tau_1 \cdot \tau_2 \\ & + \frac{\sqrt{5}}{18}f(l=0,r_1,r_2)\begin{pmatrix} 2\sqrt{5} & \lambda=0 \\ -\sqrt{5} & \lambda=1 \\ 1/\sqrt{5} & \lambda=2 \end{pmatrix} \\ & \times \sum_\mu (Y_1(1)\sigma_1)^+_{\lambda\mu}(Y_1(2)\sigma_2)_{\lambda\mu}\tau_1 \cdot \tau_2. \end{aligned} \tag{5}$$

It is known that the first term produces the coupling between GT and 1^+ SQ transitions,[5,13] while the remaining terms are expressed here by means of SD operators.[14] When $l = 1$, neglecting the terms already accounted for by Eq. (5), we can write

$$\begin{aligned} V_T^{l=1} = & \sqrt{\frac{3}{10}}f(l=1,r_1,r_2)\{[(Y_3(1)\sigma_1)_2(Y_1(2)\sigma_2)_2] \\ & + [(Y_1(1)\sigma_1)_2(Y_3(2)\sigma_2)_2]_0\}\tau_1 \cdot \tau_2 \\ & + \frac{1}{2}f(l=1,r_1,r_2)\begin{pmatrix} 7/15 & \lambda=1 \\ -7/15 & \lambda=2 \\ 2/15 & \lambda=3 \end{pmatrix} \\ & \times \sum_\mu (Y_2(1)\sigma_1)^+_{\lambda\mu}(Y_2(2)\sigma_2)_{\lambda\mu}\tau_1 \cdot \tau_2, \end{aligned} \tag{6}$$

where the first term produces the coupling between 2^- states of SD and SO modes, and the remaining terms are made up with SQ operators. Of course, we could go further but this would be of mere academic interest since we do not expect the onset of collectivity in the high angular momentum channels.

Since both $f(l = 0, r_1, r_2)$ and $f(l = 1, r_1, r_2)$ are positive, then all our calculated results for the SD and SQ modes (which include the RPA tensor correlations) can be understood clearly by invoking the formulae (5) and (6), respectively. For these SD and SQ modes, from Eqs. (5) and (6), we can see that in the $\lambda = l \pm 1$ channels, the tensor interaction is repulsive, so it shifts the centroid of the strength upward, while in $\lambda = l$ channels the tensor interaction is attractive and so it shifts the centroid of the strength downward.

In conclusion, the effects of the Skyrme tensor terms on the charge-exchange SD, SQ and some relevant SO and SH transitions in ^{90}Zr have been studied. It is found that the tensor force produces dramatic changes on the strength functions of SD and SQ transitions because of the tensor correlations: The centroid of $(Y_\ell\sigma)_{\lambda=\ell-1}$ and $(Y_\ell\sigma)_{\lambda=\ell}$ transitions are shifted dramatically upward and downward, respectively, and the centroid of $(Y_\ell\sigma)_{\lambda=\ell+1}$ transition is shifted upward. In addition, the $(Y_{\ell=\lambda-1}\sigma)_\lambda$ and $(Y_{\ell=\lambda+1}\sigma)_\lambda$ modes can be coupled by the tensor terms. This coupling shifts part of the strength of the $(Y_{\ell=\lambda-1}\sigma)_\lambda$ low-frequency mode to higher energy. The value of the shift of the centroid of the strength for $(Y_\ell\sigma)_{\lambda=\ell-1}$ transition is much larger than that for $(Y_\ell\sigma)_{\lambda=\ell+1}$ transition, it may be related to the geometrical scaling of the interaction strength. We find that the shifting and coupling effects of our Skyrme type of tensor terms could be understood qualitatively by expressing a finite range tensor force in a separable form. The coupling term $[(Y_{\ell=\lambda-1}\sigma)_\lambda(Y_{\ell=\lambda+1}\sigma)_\lambda]_0$ shifts part of the strength of the $(Y_{\ell=\lambda-1}\sigma)_\lambda$ low-frequency mode to higher energy. The repulsive terms $[(Y_\ell\sigma)_{\lambda=\ell-1}(Y_\ell\sigma)_{\lambda=\ell-1}]_0$ and $[(Y_\ell\sigma)_{\lambda=\ell+1}(Y_\ell\sigma)_{\lambda=\ell+1}]_0$ shift the centroid of the strengths of $(Y_\ell\sigma)_{\lambda=\ell-1}$ and $(Y_\ell\sigma)_{\lambda=\ell+1}$ transitions upward, while the attractive term $[(Y_\ell\sigma)_{\lambda=\ell}(Y_\ell\sigma)_{\lambda=\ell}]_0$ shifts the centroid of the strength of $(Y_\ell\sigma)_{\lambda=\ell}$ transitions downward. The RPA correlations associated with the tensor terms act in such a way that the strengths of SD and SQ transitions (especially that of the SD transitions) are distributed in a much wider energy range than in the case without tensor; moreover, the high energy part of the strength function is dominated by $(Y_\ell\sigma)_{\lambda=\ell-1}$ modes. It will certainly provide useful information for the understanding of the tensor force if, by means of experiments, one could make a decomposition of the different SD$_-$ (or SQ$_-$) components.

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