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Non-Strange Baryon Spectra and Confinement in the Constituent Quark Model *

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In the frame of the constituent quark model with screened color confinement, non-strange baryon spectra are studied by using non-relativistic and semi-relativistic Faddeev equations in momentum space. The results show that a qualitative description of the high-energy baryon spectra can be obtained, especially for states at excitation energies from 1 to 1.5 GeV. Compared with the linear confinement potential, the discrepancy between non-relativistic and semi-relativistic approaches becomes smaller in the present model with screened color confinement.

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Quantum chromodynamics (QCD), the fundamental theory of the strong interaction, supports the picture that below a certain distance-scale QCD is a weakly coupled theory with asymptotically free quark and gluon degrees of freedom, but above this scale a strong coupling regime emerges in which color is confined and chiral symmetry is broken. Low-energy phenomena, from baryon spectra to baryon-baryon interactions, belong to this scale. Although lattice QCD has been making progress on these phenomena, QCD-inspired models are still powerful tools to obtain some insight into many of the phenomena of the hadronic world. Among these QCD-inspired models, the simplest and most successful approach is undoubtedly the constituent quark model,[1−6] which relies on a three-quark Hamiltonian consisting of a relativistic or non-relativistic kinetic-energy operator, a confinement and a hyperfine interaction deduced from instantaneous pseudoscalar boson exchange (GBE) and/or one-gluon exchange (OGE), or other effective interactions.

The constituent quark models provide us with a precise description of baryon spectroscopy. However, two outstanding problems remain unsolved: one is known as the missing resonance problem, many models predict a proliferation of baryon states at excitation energies above 1 GeV, which is very different from the experimental data. Some explanations for this discrepancy have been given theoretically and experimentally. For example, Capstick and Roberts thought that most of the missing states could be too inelasic to be easily observed,[7] Vijande *et al.*[8] put forward a screened potential to explain the missing resonance; another is the correct level ordering of the positive- and negative-parity states in light baryon spectra. In experiment, the positive-parity state N(1440) (Roper resonance) lies below the pair of negative-parity states N(1535)–N(1520). The same is true for the Δ spectrum with $\Delta(1600)$ and the pair $\Delta(1620)$–$\Delta(1700)$. Many models based on OGE can not describe Roper resonance's inverted position relative to the first negative parity state. Glozman[3] uses the spin-flavor symmetry of GBE to resolve this long-standing problem. However, GBE is not great enough for baryon-baryon interaction. Desplanques *at al.*[9] put forward scalar three-body forces to cure the problem. Salamanca group suggested that the problem could be tackled by combining the effect of relativistic kinematics and the pion exchange with a sufficiently large cutoff parameter.[10,11]

In the study of baryon spectra, generally the confinement potential used increases infinitely with the increasing distance between two quarks, it is natural that this potential produces an infinite discrete baryon spectrum. However, unquenched lattice gauge calculations, after taking the light $q\bar{q}$ excitations into account, show that heavy $q\bar{q}$ confinement interaction is screened.[12] Considering the contribution of sea quarks,[13] lattice QCD suggests a screening effect when the interquark distance is increased. Unfortunately, for the heavy meson spectrum, high-energy excitation data are very poor.

Constituent quark models of baryon structure are based on the assumption of effective quark degrees of freedom so that a baryon is a three-quark color-singlet state. Therefore, for a given model of the interaction between quarks, one has to deal with a three-body problem. In principle, the three-body problem can be solved by many methods, such as the Feshbach–Rubinow variational method,[14] the stochastic variational method,[15] and the Faddeev equations.[16] In this Letter, we choose the Faddeev equations. The Faddeev equations start from a fully relativistic four-dimensional theory, many approaches are used to reduced them to three-dimensional form. We use an approximation approach developed in Ref. [17], in which the three particles are kept on their mass shells at every stage. This approach can take into account the relativistic effect of kinetic energy straightforwardly. In the previous paper, Faddeev calculations of baryon spectra have been performed in configuration space,[18,19] because the Fourier transform of the confining potential does not exist. On the other hand, in the constituent quark model, even the u and d quarks take the constituent mass, the relativis-

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tic effect is not expected to be negligible. Recently, some new methods[20,21] have been put forward to deal with the infinite confinement term in momentum-space, which makes a relativistic generalization of the Faddeev equations possible.

N and Δ spectra have been calculated by solving three-body Faddeev equations in the Goldstone-boson-exchange model[19] and in the chiral quark model with screened potential, which contains a linear confinement plus the Coulomb and spin-spin term, both screened through a Yukawa-type factor, for the non-relativistic case.[8,11] The effect of relativistic effects in the nonstrange baryon spectrum is estimated by solving the Faddeev equations[22] in the BCN (proposed by Bhaduri, Cohler and Nogami) model.[23] However, many constituent quark models prefer screened confinement and keep the long-range contribution of the Coulomb potential unchanged. In the present work, the simple constituent quark model, i.e. a modified BCN model, which contains color Coulomb, color-magnetic terms and screened linear confinement, is employed to study the nonstrange baryon spectrum with non-relativistic and semi-relativistic Faddeev equations in the momentum-space. In order to compare with experimental data, we expand our research region to about 1.5 GeV. Meanwhile, by solving the three-body problem we obtain more reliable Hamiltonian and model parameters which can reproduce the meson and high-energy baryon spectrum, and extend it to the study of nucleon-nucleon scattering and multiquark systems in a unified way.

The Faddeev equation reads

$$|\phi_i\rangle = G_0(E)t_i(E)[|\phi_j\rangle + |\phi_k\rangle], \tag{1}$$

where $|\phi_i\rangle$ is the wave-function of particle i under Faddeev decomposition and $t_i(E)$ is the solution of the Lippmann-Schwinger equation

$$t_i(E) = V_i + V_i G_0(E) t_i(E). \tag{2}$$

In the three-body center of mass frame, the propagator can be written as

$$G_0(E, p_i, q_i) = \left[E - \frac{p_i^2}{2\eta_i} - \frac{q_i^2}{2\nu_i}\right]^{-1}, \tag{3}$$

for the non-relativistic case, where p_i and q_i are the Jacobi momenta, p_i is the relative momentum between particles j and k and q_i is the relative momentum between the particle-pair jk and particle i, η_i and ν_i are the corresponding reduced masses, and

$$\begin{aligned} G_0(W_0) &= \frac{1}{W_0 - W_i(p_i, q_i) - \omega_i(q_i)}, \\ W_i(p_i, q_i) &= \sqrt{\omega^2(p_i) + q_i^2}, \quad \omega_i(q_i) = \sqrt{m_i^2 + q_i^2}, \\ \omega(p_i) &= \sqrt{m_j^2 + p_i^2} + \sqrt{m_k^2 + p_i^2}, \end{aligned} \tag{4}$$

for the relativistic case.

In the non-relativistic case, assuming that the two-body t-matrix and potential matrix V are both invariant under space shift translation, the Faddeev equations can be projected into partial waves,[24]

$$\begin{aligned} &\langle p_i q_i; l_i, \lambda_i, S_i, T_i | \phi_i^{\rm LST}\rangle \\ &= G_0(E, p_i q_i) \sum_{j\neq i} \sum_{l_j \lambda_j S_j T_j} \frac{1}{2} \int d\cos\theta \int_0^{+\infty} q_j^2 dq_j \\ &\quad \cdot t_{i,l_i,S_i,T_i}(p_i, p_i', E - q_i^2/2\nu_i) A_L^{l_i\lambda_i l_j \lambda_j}(p_i' q_i p_j q_j) \\ &\quad \cdot \langle S_i T_i | S_j T_j\rangle_{ST} \langle p_j q_i l_j \lambda_j | \phi_j^{\rm LST}\rangle, \end{aligned} \tag{5}$$

where l_i is the orbital angular momentum of the particle-pair jk, λ_i is the orbital angular momentum of particle i with respect to the pair jk, S_i and T_i are the spin and isospin of the pair jk, S and T are the total spin and isospin, L is the total orbital angular momentum, p_i' and p_j on the right-hand side of the above equation can be expressed as

$$\begin{aligned} p_i' &= \sqrt{q_j^2 + \left(\frac{\eta_i}{m_k}\right)^2 q_i^2 + \frac{2\eta_i}{m_k} q_i q_j cos\theta}, \\ p_j &= \sqrt{q_i^2 + \left(\frac{\eta_j}{m_k}\right)^2 q_j^2 + \frac{2\eta_j}{m_k} q_i q_j cos\theta}, \end{aligned}$$

θ is the angle of q_i and q_j.

$$\begin{aligned} &A_L^{l_i\lambda_i l_j\lambda_j}(p_i' q_i p_j q_j) \\ &= \frac{1}{2L+1} \sum_{M m_i m_j} C^{l_i,\lambda_i,L}_{m_i,M-m_i,M} C^{l_j,\lambda_j,L}_{m_j,M-m_j,M} \Gamma_{l_i,m_i} \\ &\quad \cdot \Gamma_{l_j,m_j} \Gamma_{\lambda_i,M-m_i} \Gamma_{\lambda_j,M-m_j} \cos[-M(\widehat{\boldsymbol{q}_j, \boldsymbol{q}_i}) \\ &\quad - m_i(\widehat{\boldsymbol{q}_i, \boldsymbol{p}_i'}) + m_j(\widehat{\boldsymbol{q}_j, \boldsymbol{p}_j})], \end{aligned} \tag{6}$$

$$\Gamma_{l,m} = \begin{cases} \frac{(-)^{(l+m)/2}\sqrt{(2l+1)(l+m)!(l-m)!}}{2^l((l+m)/2)!((l-m)/2)!}, \\ \qquad l-m = \text{even}, \\ 0, \qquad l-m = \text{odd}, \end{cases} \tag{7}$$

$$\begin{aligned} \langle S_i T_i | S_j T_j\rangle_{ST} &= (-)^{S_j+\sigma_j-S} \sqrt{(2S_i+1)(2S_j+1)} \\ &\quad \cdot W(\sigma_j \sigma_k S \sigma_i; S_i S_j) \\ &\quad \cdot (-)^{T_j+\tau_j-S} \sqrt{(2T_i+1)(2T_j+1)} \\ &\quad \cdot W(\tau_j \tau_k S \tau_i; T_i T_j), \end{aligned}$$

since the wave-function is color antisymmetric, l_i, S_i and T_i should satisfy the relationship $(-)^{l_i+S_i+T_i} = 1$.

For three identical particles, $|\phi_i\rangle = |\phi_j\rangle = |\phi_k\rangle$, making the change of variables

$$p_i = b\frac{1+x_i}{1-x_i}, \quad q_i = b\frac{1+y_i}{1-y_i},$$

where $b(\text{fm}^{-1})$ is a scale parameter, the variables x_i and y_i run from -1 to 1, so Eq. (5) becomes

$$\begin{aligned} &T^\alpha(x_i, y_i, E) \\ &= G_0(E, x_i, y_i) \sum_\beta \int_{-1}^{+1} d\cos\theta \int_{-1}^{+1} dy_j \end{aligned}$$

$$\cdot \frac{2b}{(1-y_j)^2} t_\alpha\left(x_i, x_i', E - \frac{1}{2\nu_i}\left(b\frac{1+y_i}{1-y_i}\right)^2\right)$$
$$\cdot B^{\alpha\beta}(y_i, y_j, \cos\theta, E) T^\beta(x_j, y_j, E), \quad (8)$$

$$B^{\alpha\beta}(y_i, y_j, \cos\theta, E) = A_L^{l_i\lambda_i l_j\lambda_j}(x_i', y_i, x_j, y_j) \cdot \langle S_i T_i | S_j T_j \rangle_{ST} \left(b\frac{(1+y_j)}{(1-y_j)}\right)^2,$$

where α and β stand for the quantum numbers l_i, λ_i, S_i, T_i and l_j, λ_j, S_j, T_j, respectively.

Expand the two-body and three-body amplitude in terms of Legendre polynomials as

$$t_\alpha(x_i, x_i', e) = \sum\nolimits_n P_n(x_i) t_{\alpha,n}(x_i, e), \quad (9)$$

$$T_\alpha(x_i, y_i, E) = \sum\nolimits_n P_n(x_i) T_{\alpha,n}(y_i, E), \quad (10)$$

therefore, Eq. (8) reduces to a set of integral equations in one variable,

$$T_{\alpha,n}(y_i, E) = \sum_{\beta,m} \int_{-1}^{+1} dy_j B_{nm}^{\alpha\beta}(y_i, y_j, E) T_{\beta,m}(y_j, E). \quad (11)$$

Expanding the potential as

$$V_\alpha(x_i, x_i') = \sum\nolimits_n P_n(x_i) V_{\alpha,n}(x_i'), \quad (12)$$

and using Eq. (9), Eq. (2) reduces to a set of linear equations,

$$t_{\alpha,n}(x_i', e) = V_{\alpha,n}(x_i') + \sum\nolimits_k a_{\alpha,nk}(e) t_{\alpha,k}(x_i', e), \quad (13)$$

with

$$a_{\alpha,nk}(e) = \int_{-1}^{+1} 2b^2 \frac{(1+x_i'')^2}{(1-x_i'')^4} V_{\alpha,n}(x_i'') \cdot G_0(e, x_i'') P_k(x_i'') dx_i'',$$

$$V_{\alpha,n}(x_i') = \frac{2n+1}{2} \int_{-1}^{+1} dx_i P_n(x_i) V_{\alpha,n}(x_i, x_i'), \quad (14)$$

$$V_\alpha(x_i, x_i') = \frac{2}{\pi} \int_0^\infty r_i^2 dr_i j_{l_i}\left(b\frac{1+x_i}{1-x_i} r_i\right) \cdot V_i(r_i) j_{l_i}\left(b\frac{1+x_i'}{1-x_i'} r_i\right), \quad (15)$$

$j(x)$ is the spherical Bessel function.

In solving the set of linear equations (13), where functions $V_{\alpha,n}(x_i')$ and $a_{\alpha,nk}(e)$ enter as input, a much larger number of the Legendre expansion (about 200 Legendre polynomials) is needed, since the momentum-space representation of potential Eq. (15) oscillates very rapidly. After obtaining $t_{\alpha,n}(x_i', e)$, Eq. (11) is solved to obtain the eigen-energies of the system.

Here we only give a brief introduction of the method, the details can be seen in Ref. [22].

In this study, our main interest is to explore the screening effect of confinement for non-strange baryon spectra. Firstly, we repeat the calculation of Ref. [22] by using the BCN model to check the correctness and stability of our calculations. Our results are in agreement with those of Ref. [22] with an accuracy of 10 MeV. Based on the above calculations, we replace the linear confinement by the screening confinement, and keep the parameters unchanged; the new potential is written as

$$V_i(r_i) = \frac{1}{2}\left(\frac{-\kappa}{r_i} - D + \frac{\kappa}{m^2} e^{-r_i/r_0} \sigma_j \cdot \sigma_k\right) + a_c(1 - e^{-\mu r_i}), \quad (16)$$

where $\kappa = 102.67$ MeV, $r_0 = 0.4545$ fm, $\mu = 0.7$ fm^{-1}, taken from Ref. [25]. The effective confinement strength $a_c = 1200$ MeV is determined by the mass difference of $\Delta - N$, the parameter D is decided by reproducing the mass of the nucleon. Since D has no effect on the excitation energies, we let $D = 1188.8$ MeV for non-relativistic theory and $D = 1018.5$ MeV for relativistic theory.

Using the above screened potential, the calculated spectra of N and Δ for the relativistic and non-relativistic theories (l_i and λ_i are restricted up to 3) are shown in Tables 1 and 2.

Table 1. $N(1/2^+)$ ($L = 0$) spectra (in MeV) for relativistic and non-relativistic theories.

$l_i = \lambda_i = 0$	l_i, λ_i up to 1	l_i, λ_i up to 2	l_i, λ_i up to 3
non-relativistic			
960	945	939	939
1613	1603	1580	1578
-	1753	1748	1744
2023	2008	1973	1966
-	2098	2088	2076
relativistic			
963	943	940	939
1503	1490	1478	1478
-	1743	1735	1730
1903	1893	1870	1868
-	2068	2049	2038

Table 2. The same as Table 1 but for Δ $(3/2^+)(L = 0)$.

$l_i = \lambda_i = 0$	l_i, λ_i up to 2	$l_i = \lambda_i = 0$	l_i, λ_i up to 2
non-relativistic		relativistic	
1258	1256	1263	1263
1796	1790	1737	1732
2158	2134	2093	2083
-	2348	2360	2338

From Tables 1 and 2, we find that the masses of the baryons decrease with the increase of the included three-body configurations, especially for the higher excitations. For $\Delta(3/2^+)$, there is no contribution for odd l_i because of the restriction of the Pauli principle. The mass splitting of N and Δ is 317 MeV for non-relativistic theory, and 324 MeV for the relativistic case.

Figure 1 shows N and Δ spectra of our calculation, compared with the previous calculation without screening in the non-relativistic theory. It is clear that the results with screening potential are in better agreement with the experimental data. We should emphasize that, after replacing linear confinement by the screened form, the number of energy levels be-

comes numerable. This result is consistent with that of Ref. [8] in which the Coulomb and OGE interactions are both screened. However, in the range from 1 to 1.5 GeV, the number of excited states from the model calculation is still more than that from experiments (see Figs. 2 and 3).

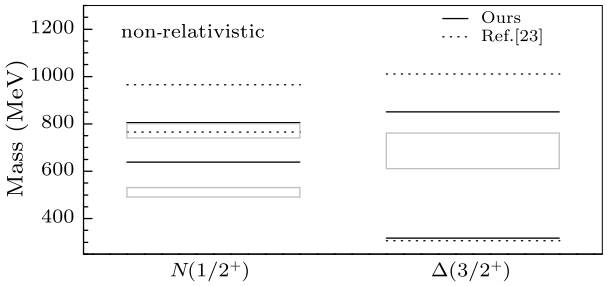

Fig. 1. $N(1/2^+)$ and $\Delta(3/2^+)$ spectra below 1 GeV (with $L = 0$) for the non-relativistic theory (in MeV). Size of the shaded regions stands for the experimental uncertainty.[26]

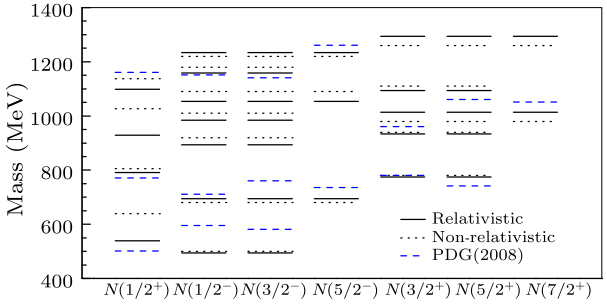

Fig. 2. N spectrum for the non-relativistic and relativistic theories. The thicker line stands for the experimental data.[27]

As can be seen from Figs. 2 and 3, as a whole, the description provided by the relativistic model is of similar quality to that of the non-relativistic one and in some respects even better. Generally the relativistic kinetic effect will lower the excited energies 10–50 MeV. For the $N(1440)$ state, the relativistic effect reduces the mass from 1578 MeV in the non-relativistic case to 1479 MeV, which is very close to the experimental value. In Ref. [22], the relativistic effect brought about 200 MeV changes.[22] However, the ordering between state $N(1440)$ and state $N(1535)$ is still inverted in both the relativistic and non-relativistic models, although the discrepancy is small (about 45 MeV). Here we keep most of the model parameters unchanged, the discrepancy would become smaller by adjusting the parameters properly as Ref. [10].

For states $N(5/2^-)$, $N(3/2^+)$, $N(5/2^+)$ and $N(7/2^+)$, our results are consistent with the experimental data, the largest difference is about 25 MeV, although there are more excited states waiting for experimental confirmation. For negative-parity groups $N(1/2^-)$ and $N(3/2^-)$, states $N(1650)$, $N(2090)$ and $N(2080)$ can be reproduced well by considering nucleon orbit ($L \neq 0$) and spin excitation ($S = 3/2$); for states $N(1520)$, $N(1535)$ and $N(1700)$, our results are below the experimental data by about 100 MeV, which is similar to that of the previous calculation.[21] How to improve the description is worth further studying. For Δ states, our calculation can describe the experimental data at excitation energy about 960–1010 MeV very well, except for $\Delta(1930)5/2^-$, which may be reproduced by considering higher orbit excitation ($L = 4$).

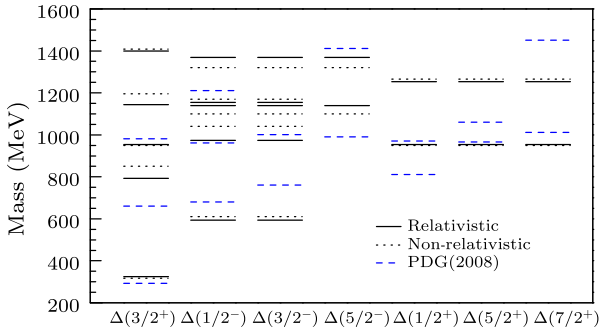

Fig. 3. The same as Fig. 2 but for the Δ spectrum.

In summary, by introducing color screening to the confinement, the constituent quark model gives a qualitative description of the non-strange baryon spectrum at excitation energies from 1 to 1.5 GeV in the framework of non-relativistic and semi-relativistic Faddeev equations in the momentum-space. The relativistic and non-relativistic theories give very similar results, which is very different from the work using the linear confinement potential without screening. With the introducing of color screening, the missing resonance problem is alleviated. However the correct level ordering of the positive- and negative-parity states is still out of reach in the present approach. Multiquark and/or hybrid states may be the way out.

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