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Generation of Efficient Dispersive Waves in Photonic Crystal Fibers *

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Based on the generalized nonlinear Schrödinger equation, we investigate efficient dispersive wave (DW) generation in a photonic crystal fiber (PCF) by numerical simulation and discuss a way to control DW generation by using an initial input pulse chirp. It is shown that efficient red-shifted DW generation can be obtained in a PCF with negative dispersion slopes. The energy contained in the DWs is considerably decreased for both positively and negatively chirped pulses at the fiber output. This provides us with an opportunity to conveniently and efficiently manipulate the DW generation by controlling the pre-chirp of the soliton. Moreover, we also show that forth- and higher-order dispersion terms play little part in deciding the evolution of DWs.

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Up to now, the physical mechanism underlying supercontinuum generation in photonic crystal fibers (PCFs) has been well understood.[1]. Nonetheless, knowing how to effectively control the excited frequency with particular desired properties is still a challenging work. For example, it is difficult (and expensive) to generate laser light at wavelengths beyond those offered by the small canon of available lasers. The efficient transfer of laser energy to new wavelengths is therefore a highly attractive prospect.[2] Recently, Skryabin *et al.*[3] achieved the cancellation of soliton self-frequency shift (SSFS)[4] in PCFs based on the interaction of solitons and dispersive waves (DWs). The soliton will emit so-called DWs when it is perturbed by higher-order dispersion.[5] Depending on the phase-matching condition,[6,7] the wavelength of DW can be either blue-shifted or red-shifted with respect to the soliton central wavelength. This radiation is also known as Cherenkov radiation,[6] or as nonsolitonic radiation (NSR).[7] According to the phase-matching condition, we can estimate the spectral position of the DW.

It is well known that the main contribution to the position of DW is due to the third order dispersion term.[8,9] If we neglect the forth- and higher-order dispersion, the wavelength of DW is governed by the group-velocity dispersion (GVD) parameter β_2 and its slope β_3.[6] For a soliton propagating in the anomalous-dispersion region such that $\beta_2 < 0$, if $\beta_3 > 0$, the DW is emitted at a wavelength shorter than that of the soliton. However, in the event of $\beta_3 < 0$, the DW is emitted at a wavelength longer than that of the soliton. Meanwhile, as the nonlinear coefficient of PCFs increases, the required propagation distance which realizes the energy transfer is reduced. On the other hand, if the input pulse propagates in the normal dispersion region, the initial dynamics are dominated by the interaction of SPM and normal GVD; this leads to significant temporal broadening and rapid decrease of peak power over the first few distances of propagation, thereby, the energy transfer from the input pulse to the DWs can hardly take place. In conventional fibers, only blue-shifted DW is observed because the slope of β_2 is always positive for commonly used frequencies.[3] The advent of PCFs provides an opportunity for the observation of red-shifted DWs due to their guidance properties and the ability to engineer their GVD.[2,10–12] In this Letter, we demonstrate efficient red-shifted DW generation in PCFs with negative dispersion slopes. We also show that appropriately controlling the pre-chirp of solitons can be applied to conveniently and efficiently manipulate DW generation in PCFs.

We assume that dynamics of the dimensionless amplitude $A(\xi,\tau)$ of the fundamental fiber mode is governed by the generalized nonlinear Schrödinger equation (GNSE)[13]

$$\partial_\xi A = iD(i\partial_\tau)A + i\gamma A\left[\int_{-\infty}^{\infty} R(\tau')|A(\xi,\tau-\tau')|^2 d\tau'\right]. \tag{1}$$

The dispersion operator in Eq. (1) is given by

$$D(i\partial_\tau) = i\sum_{k\geq 2} \frac{i^k T_0^{2-k}\beta_k}{k!|\beta_2|}\frac{\partial^k A}{\partial\tau^k}, \tag{2}$$

where τ is the time in the reference frame moving with group velocity and measured in units of T_0,

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$\tau = (t - \beta_1 z)/T_0$, T_0 is half-width at $1/e$ intensity point and t is the physical time, $\xi = z/L_D$, where z is the distance along the fiber and $L_D = T_0^2/|\beta_2|$ is the GVD length; β_k is the kth order dispersion coefficients associated with the Taylor series expansion of the propagation constant $\beta(\omega)$ about a carrier frequency ω_0.

The second term on the right hand side of Eq. (1) models the nonlinear effects: $\gamma = n_2\omega_0/cA_{\rm eff}$ is the usual nonlinear coefficient, where n_2 is the nonlinear refractive index and $A_{\rm eff}$ is the mode effective area, both evaluated at the carrier frequency ω_0. The response function $R(t) = (1-f_R)\delta(t)+f_R h_R(t)$ includes both instantaneous electronic and delayed Raman contributions, with $f_R = 0.18$ representing the Raman contribution, $h_R(t)$ is the Raman response function of silica fiber and can be approximated by the expression $h_R(t) = \exp(-t/\tau_2)\sin(t/\tau_1)(\tau_1^2+\tau_2^2)/\tau_1\tau_2^2$, where the time constants τ_1 and τ_2 are chosen to be 12.2 fs and 32 fs, respectively.[14]

Numerical solutions of Eq. (1) are obtained by using the symmetrized split-step Fourier method.[13] In the process of calculation, we introduce the normalized time and distance variables defined as

$$\tau = \frac{T}{T_0} = \frac{t - z/v_g}{T_0}, \quad \xi = \frac{z}{L_D},$$
$$L_D = \frac{T_0^2}{|\beta_2|}, \quad N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|}, \tag{3}$$

where the integer values of N are related to the soliton order. The input pulses are assumed to have the form $A(0,t) = \mathrm{sech}(t/T_0)\exp(-iCt^2/2T_0^2)$, where C is the parameter representing the initial linear frequency chirp.

We consider propagation in a highly nonlinear PCF with core diameter 1.2 μm.[3,15] The two ZDWs are located at 600 nm and 1310nm, respectively. The dispersion between the two ZDWs is anomalous and outside the region it is normal. Numerical simulations are carried out by setting $\lambda_0 = 1.20\,\mu$m, $\beta_2 = -75\,\mathrm{ps}^2/\mathrm{km}$, $\beta_3 = -0.5\,\mathrm{ps}^3/\mathrm{km}$, $T_0 = 30$ fs and $N = 1$. We note that the main contribution to the position of DWs is due to the third order dispersion term.[8,9] The reason we select the input wavelength close to the ZDW located in the infrared region is that DW generation is easier.[16] To gain a physical understanding of the effects, the propagation loss is neglected. In the mean time, we have also neglected here the self-steepening term since it has no qualitative and only a small quantitative effect on the results.[15]

Figure 1(a) illustrates the spectrum evolution along with the fiber for an input chirp-free fundamental soliton. From the figure, we can clearly see that the radiation in the red-shifted band emitted by the soliton begins to emerge at $z \cong 28L_D$. For $z > 32L_D$, the SSFS is cancelled by the spectral recoil effect from the radiation. It is worth noting that the DW emerges much earlier at larger frequency shifts and a much weaker intensity immediately along propagation which is invisible in the linear scale. At the fiber output, the energy focuses on two regions, round the ZDW and the dispersive wavelength, depending on the phase-matching condition. When the initial input pulse is not the fundamental soliton provided that $N > 1$, simulations not shown here indicate that the DWs have the similar qualitative behaviour. The difference is that the required propagation distance is reduced due to enhanced nonlinear effects.

Nevertheless, the initial frequency chirp of the input soliton can lead to drastic changes in the dynamics of DW generation. This is illustrated in Fig. 1(b), which shows the spectra of solitons with $C = 0$, $C = 0.3$ and $C = -0.3$ at $z = 32L_D$, respectively. For an input chirp-free soliton, the DW has already been generated and the spectrum intensity is higher than that of the soliton due to the energy transfer. However, for a soliton with positive chirp $C = 0.3$, the DW has been generated but the spectral intensity is low, which means that the process of energy transfer is just beginning. Furthermore, when the chirp is negative, the energy transfer has not begun and DW has not been generated at all.

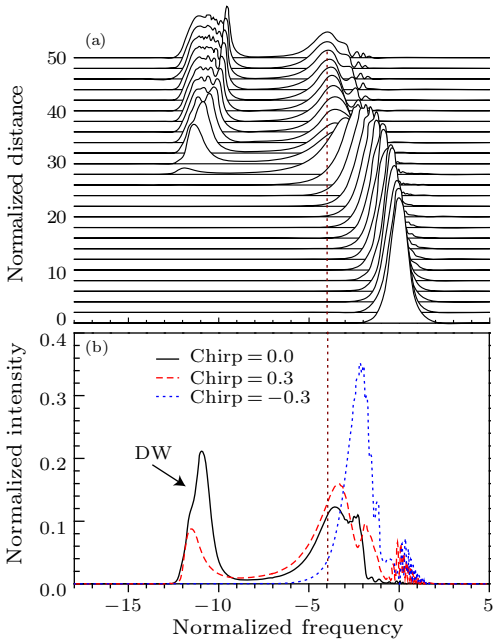


Fig. 1. Spectrum evolution along with propagation distance for an input chirp-free fundamental soliton (a). Spectra for input solitons with $C = 0$, $C = 0.3$ and $C = -0.3$ at $z = 32L_D$, respectively (b). The vertical dashed lines mark the zero GVD frequency.

To obtain more straightforward information of the energy distribution at the PCF output, we calculate the percentage of the energy contained in the DW defined as follows: DW energy $\eta(\%) = (E_{\rm dis}/E_{\rm total}) \times 100\%$. Calculated η as a function of the initial fre-

quency chirp at $z = 50L_D$ without (red lines) and with (blue lines) the inclusion of fourth-order dispersion (FOD) is shown in Fig. 2. In our consideration, we first take no account of the effects of forth-order dispersion. For positively chirped solitons, a noteworthy feature of Fig. 2 is that there are three stages of energy contained in the DW varying with initial frequency chirp. In the first stage, the DW energy decreases slowly as the effect of chirp is slight. Nevertheless, the DW energy decreases drastically when the initial chirp attains a certain value. That is to say, even a relatively small change of chirp affects the DW energy considerably. Thus, this provides us with a convenient and efficient method to control the percentage of energy contained in the DW. In the third stage, the DW emitted by the soliton does not emerge. For negatively chirped solitons, the DW energy shows similar behavior except that the change is more rapid. From the figure, we can see that the DW contains about 66.0% of the input energy at the fiber output if the input soliton is chirp free. The efficiency of energy transfer is very high. However, no matter whether the chirp is positive or negative, the percentage of the energy contained in the DW will change from 66.0% to zero at the fiber output. In order to elucidate the effects of FOD on DWs, we assume $\beta_4 = 8.1 \times 10^{-4}\,\mathrm{ps}^4/\mathrm{km}$ and calculate the variation of energy contained in the DW with the inclusion of FOD. From Fig. 2, we can clearly see that the variation of the energy of DWs with the inclusion of FOD changes more slowly compared to the variation without the inclusion of FOD for both positive and negative chirps. It is readily seen that the FOD plays little role in deciding the evolution of DWs.

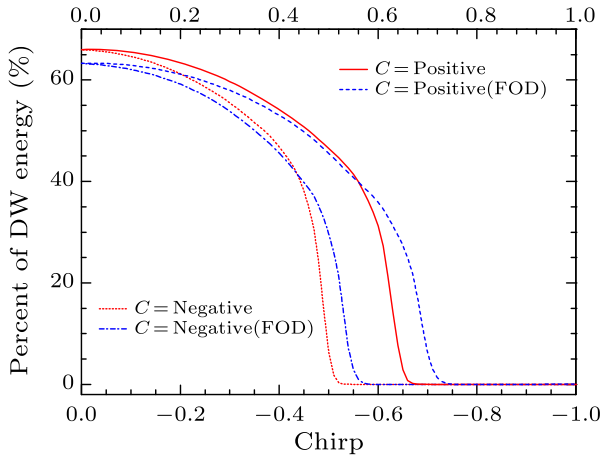


Fig. 2. Variation of the energy contained in the dispersive wave at $z = 50L_D$ with the initial frequency chirp of the soliton without (red lines) and with (blue lines) the inclusion of FOD. The top (bottom) axis is marked in positive (negative) chirp.

The reason for this behaviour can be understood as shown in Fig. 3. It is well known that a soliton propagating in a perturbed regime experiences a dynamical evolution of its parameters. If the dispersion slope is negative, the absolute value of GVD ($|\beta_2|$) decreases as the soliton experiences a continuous shift to longer wavelengths due to Raman induced frequency shift (RIFS). The decrease of $|\beta_2|$ results in the narrowing of soliton width and increase of its amplitude[17] in order to maintain the properties of soliton. When it approaches the ZDW, then the soliton peak intensity will increase exponentially (Fig. 3(a)). It may be stressed that the fundamental condition that allows the energy transfer from the soliton to the DW is given by the superposition of the soliton spectrum with the spectral region[16,18] decided by the phase-matching condition. The DW is emitted around the point of strong temporal compression of the soliton when the long-wavelength tail of the spectrum has deeply extended even into the normal dispersion region. It is obvious from a comparison of Fig. 3 and Fig. 1(a) that notable DW generation starts at the propagation distance which corresponds to the maximum peak intensity.

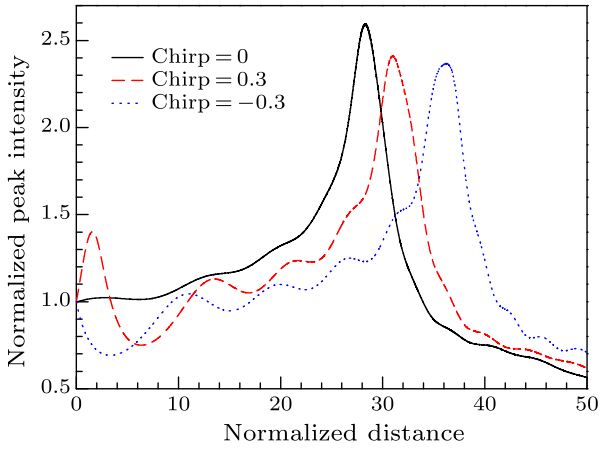


Fig. 3. Variation of normalized peak intensity as a function of normalized distance for input solitons with different initial frequency chirps.

For chirped solitons, the soliton eigenvalues which represent the ultimate soliton amplitudes are reduced for both signs of the chirp.[19−21] As the chirp increases the eigenvalues move toward zero, meaning that the solitons are less energetic. It is clear that for large absolute values of the initial chirp, no solitons or DW are produced at all. As a consequence, the process of approaching the ZDW for chirped solitons requires a longer propagation distance. Thereby, the energy transfer from soliton to DW for chirped solitons is deferred in contrast to that for the chirp free soliton. For negative values of chirp, solitons begin to broaden immediately, and the energy transfer is deferred further.

It is interesting to note that if the fiber is long enough and the fiber loss is negligible, the energy will continue to transfer from the soliton to the DW due to perturbation by third-order dispersion even if the

RIFS is suppressed by the spectral recoil effect. This means that we can achieve a very efficient conversion of energy from the soliton to the DW. The reduction of energy conversion induced by frequency chirp can be easily overcome by simply increasing the fiber length.

In conclusion, we have numerically investigated the propagation of fundamental solitons in PCFs with a negative dispersion slope based on the GNSE. Simulation results show that the efficiency of energy conversion from the soliton to the DW is very high. The initial chirp provides us with the opportunity to conveniently and efficiently control the energy transfer process. The efficient transfer of laser energy to new wavelengths therefore has high potential in many applications.

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