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Spin Accumulation in a Double Quantum Dot Aharonov–Bohm Interferometer *

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We investigate the spin accumulation in a double quantum dot Aharonov–Bohm (AB) interferometer in which both the Rashba spin–orbit (RSO) interaction and intradot Coulomb interaction are taken into account. Due to the existence of the RSO interaction, the electron, flowing through different arms of the AB ring, will acquire a spin-dependent phase factor in the tunnel-coupling strengths. This phase factor will induce various interesting interference phenomena. It is found that the electrons of the different spin directions can accumulate in the two dots by properly adjusting the bias and the intradot level with a fixed RSO interaction strength. Moreover, both the magnitude and direction of the spin accumulation in each dot can be conveniently controlled and tuned by the gate voltage acting on the dot or the bias on the lead.

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The quantum dot (QD), in which the spin coherence time for the electron is relatively long,[1] is considered as an ideal candidate for the qubit to realize quantum computing in the future. The challenging issue in mesoscopic QD systems is how to control electron spins.[2] Traditionally, the way to manipulate the spin polarization is coupling with ferromagnetic (FM) leads or using a magnetic field.[3,4] However there exist some difficulties to realize spin-polarized transport in experiments by using these two methods in a mesoscopic QD system.[5] The spin-polarized currents in the FM lead are difficult to inject into a semiconductor due to a large resistivity mismatch, and for the second proposal to succeed, one needs a very strong external magnetic field confined to a small region of a QD.

Therefore, an alternative method, such as using an intrinsic property of the QD to manipulate and control the electron spin, is desirable. In a semiconductor mesoscopic system, the Rashba spin–orbit (RSO) interaction comes into play by introducing an electric potential to destroy the symmetry of space inversion.[6,7] It can couple the spin degree of freedom to its orbital motion so that it is possible to control the electron spin in terms of external electric fields or gate voltages. Recently, Sun and Xie theoretically predicted that the direction and the strength of the spin polarization in QD can be well controlled and manipulated by the bias and RSO interaction.[8] Sinova *et al.*[9] found that a substantial amount of dissipationless quantum spin current could be generated by a co-action of electric field and RSO interaction. There are also many theoretical and experimental works concerning the spin splitter[10] and the possibility of generation of the pure spin current[11] by means of RSO interaction.

In this Letter, we propose a scheme to polarize and manipulate the electronic spin in an AB interferometer with an interacting QD embedded in each arm. Based on the Keldysh nonequilibrium Green function, the spin accumulation in each QD has been calculated. It is shown that not only the spin accumulation of each QD can be produced with a fixed RSO interaction, but also the direction and the strength of the spin accumulation can be conveniently modulated by varying the gate voltage or the bias.

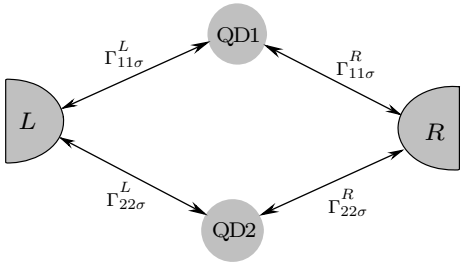

Fig. 1. Schematic setup of the AB interferometer with two QDs connected to two leads.

We consider a double-dot AB interferometer, connected to the left and right leads as schematically shown in Fig. 1. For simplicity, it is assumed that only one spin-degenerate energy level in each dot is relevant. The interdot Coulomb interaction is assumed to be negligibly small as compared with the intradot interaction. We also assume that the tunnel junction between the two dots is sufficiently thick and the direct interdot tunnelling coupling is neglected. The RSO interaction, which gives rise to a spin-dependent phase factor in the hopping matrix element between the lead and QD,[12] is supposed to exist only inside the QD. The full system can be described by the following Hamiltonian:

$$H_{\rm total} = \sum_{k,\sigma} \sum_{\alpha=L,R} \varepsilon_{k\alpha} c^+_{k\alpha\sigma} c_{k\alpha\sigma} + \sum_{i\sigma} \varepsilon_{i\sigma} d^+_{i\sigma} d_{i\sigma}$$

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$$+\sum_i U_i d_{i\uparrow}^+ d_{i\uparrow} d_{i\downarrow}^+ d_{i\downarrow}$$
$$+\sum_{ki\sigma}\sum_{\alpha=L,R}\left(t_{\alpha i\sigma} c_{k\alpha\sigma}^+ d_{i\sigma} + \text{H.c.}\right), \quad (1)$$

where $c_{k\alpha\sigma}^+$ ($c_{k\alpha\sigma}$) is the creation (annihilation) operator of electron in the lead α with the wave vector $\boldsymbol{k}$ and spin σ, $\varepsilon_{k\alpha}$ denotes the corresponding single particle energy. $d_{i\sigma}^+(d_{i\sigma})$ represents the creation (annihilation) operator of electron with the energy level $\varepsilon_{i\sigma}$ and spin σ in dot i, U_i describes the Coulomb interaction energy in QDi, $t_{\alpha i\sigma}$ describes the dot-lead tunnelling coupling which is assumed to be independent of k and takes the following forms: $t_{L1\sigma} = t_{L1}e^{-i\sigma\varphi_{R1}/2}$, $t_{L2\sigma} = t_{L2}e^{-i\sigma\varphi_{R2}/2}$, $t_{R1\sigma} = t_{R1}e^{i\sigma\varphi_{R1}/2}$, and $t_{R2\sigma} = t_{R2}e^{i\sigma\varphi_{R2}/2}$. The phase factor φ_{Ri} comes from the RSO interaction in dot i. In the following calculation, we define the line-width matrix elements as $\Gamma_{ij\sigma}^{\alpha} = 2\pi\sum_k t_{\alpha i\sigma}t_{\alpha j\sigma}^*\delta(\varepsilon-\varepsilon_{k\alpha})$, and then the matrix $\boldsymbol{\Gamma}_{\sigma}^{\alpha}$ can be written as

$$\boldsymbol{\Gamma}_{\sigma}^{L(R)} = \begin{pmatrix} \Gamma_1^{L(R)} & \mathcal{A} \\ \mathcal{B} & \Gamma_2^{L(R)} \end{pmatrix}, \quad (2)$$
$$\mathcal{A} = \sqrt{\Gamma_1^{L(R)}\Gamma_2^{L(R)}}e^{\pm i\varphi_\sigma/2}$$
$$\mathcal{B} = \sqrt{\Gamma_1^{L(R)}\Gamma_2^{L(R)}}e^{\mp i\varphi_\sigma/2}$$

where Γ_i^{α} is short for Γ_{ii}^{α}. In Eq. (2), the spin-dependent phase factor $\varphi_\sigma = -\sigma(\varphi_{R1}-\varphi_{R2})$ with $\varphi_R = \varphi_{R1}-\varphi_{R2}$.

By using the standard Keldysh nonequilibrium Green function method, the spin-up and spin-down electronic occupation number $n_{i\sigma}$ in dot i can be calculated self-consistently by

$$n_{i\sigma} = -i\int\frac{d\omega}{2\pi}G_{i\sigma,i\sigma}^{<}(\omega), \quad (3)$$

where the lesser Green function $\boldsymbol{G}_{\sigma}^{<}(\omega)$ and the retarded Green function $\boldsymbol{G}_{\sigma}^{r}(\omega)$ are the Fourier transforms of the lesser Green's function $\boldsymbol{G}_{\sigma}^{<}(t)$ and retarded $\boldsymbol{G}_{\sigma}^{r}(t)$, respectively. The matrix element is defined as $G_{ij\sigma}^{<}(t) = i\theta(t)\langle d_{j\sigma}^+(0)d_{i\sigma}(t)\rangle$ and $G_{ij\sigma}^{r}(t) = -i\theta(t)\langle\{d_{i\sigma}(t), d_{j\sigma}^+(0)\}\rangle$ with $i,j = 1,2$ for spin σ, respectively.

By using the equation of motion approach and adopting the Hartree–Fock approximation,[13] one can obtain the retarded Green's function $\boldsymbol{G}_{\sigma}^{r}(\omega)$ as

$$\boldsymbol{G}_{\sigma}^{r}(\omega) = \left[1-\boldsymbol{g}_{\sigma}^{r}(\omega)\boldsymbol{\Sigma}_{\sigma}^{r}\right]^{-1}\boldsymbol{g}_{\sigma}^{r}(\omega), \quad (4)$$

where the matrix element of the retarded self-energy $\boldsymbol{\Sigma}_{\sigma}^{r}$, originating from the dot-lead coupling, is given by $\Sigma_{ij\sigma}^{r} = -\frac{i}{2}(\Gamma_{ij\sigma}^{L}+\Gamma_{ij\sigma}^{R}) = -\frac{i}{2}\Gamma_{ij\sigma}$ in the wide-band limit, and $\boldsymbol{g}_{\sigma}^{r}(\omega)$ is the Green's function for the isolated double quantum dot (DQD). The inverse of matrix elements of $\boldsymbol{g}_{\sigma}^{r}(\omega)$ can be expressed as

$$\left[g_{\sigma}^{r}(\omega)^{-1}\right]_{ii} = \frac{(\omega-\varepsilon_{i\sigma})(\omega-\varepsilon_{i\sigma}-U_i)}{\omega-\varepsilon_{i\sigma}-U_i+U_i n_{i\bar{\sigma}}}, \quad (5)$$

$$\left[g_{\sigma}^{r}(\omega)^{-1}\right]_{i\bar{i}} = 0. \quad (6)$$

Within the same truncating approximation as that of the retarded Green functions, the expression of lesser Green function $\boldsymbol{G}_{\sigma}^{<}(\omega)$ can be written as

$$\boldsymbol{G}_{\sigma}^{<}(\omega) = \boldsymbol{G}_{\sigma}^{r}(\omega)\boldsymbol{\Sigma}_{\sigma}^{<}\boldsymbol{G}_{\sigma}^{a}(\omega), \quad (7)$$

where the advanced Green's function $\boldsymbol{G}_{\sigma}^{a}(\omega)$ is the Hermite conjugate of the retarded $\boldsymbol{G}_{\sigma}^{r}(\omega)$, and the lesser self-energy is $\boldsymbol{\Sigma}_{\sigma}^{<} = i(f_L\boldsymbol{\Gamma}_{\sigma}^{L}+f_R\boldsymbol{\Gamma}_{\sigma}^{R})$, in which $f_\alpha(\omega) = \left[e^{(\omega-\mu_\alpha)/k_BT}+1\right]^{-1}$ is the Fermi–Dirac distribution function on the lead $\alpha(\alpha = L,R)$.

Thus the electronic occupation number $n_{i\sigma}$ can be calculated self-consistently by Eq. (3) and the spin accumulation in dot i is given by

$$\Delta n_i = n_{i\uparrow} - n_{i\downarrow}. \quad (8)$$

In the following numerical analysis, we suppose that the left and right leads are made from the identical material and assume constant effective coupling strength in the wide-band limit, $\Gamma_1^{\alpha} = \Gamma_2^{\alpha} = \Gamma(\alpha \in L,R)$ for both the spin-up and spin-down orientations and take Γ as energy units. When the external voltage V is applied between the two leads, the corresponding chemical potential of the lead is taken as $\mu_L = -\mu_R = V/2$. For simplicity, we also assume that the intradot Coulomb interactions in the two dots are the same, i.e. $U_1 = U_2 = U$.

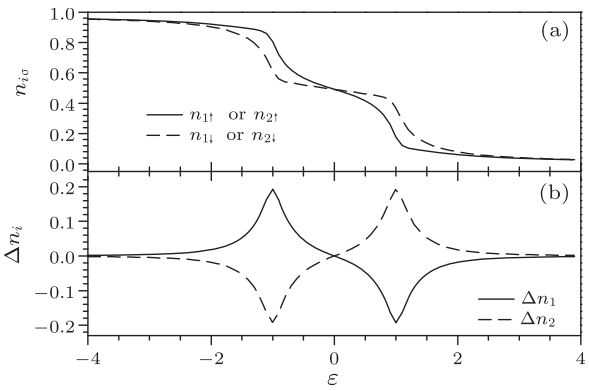

Fig. 2. Behaviour of $n_{i\sigma}$ and Δn_i versus intradot level ε with the parameters $V = 2$, $\varphi_R = \pi/2$, $K_BT = 0$ and $U = 0$.

Figure 2 shows the occupation number $n_{i\sigma}$ and the spin accumulation Δn_i versus the intradot level, which can be tuned by the gate voltage acting on the dot, at a fixed bias $V = 2$ and $\varphi_R = \pi/2$. Here we assume that the intradot level in each dot is equal, i.e. $\varepsilon_1 = \varepsilon_2 = \varepsilon$, and both of them can be tuned synchronously. The occupation number $n_{i\sigma}$ and the spin accumulation $\Delta n_{i\sigma}$ have the following features: (1) The occupation number of each dot decreases with the intradot level increasing from negative value to positive value [see Fig. 2(a)]. When the intradot level passes the chemical potential of two leads, the occupation numbers of each spin component, $n_{i\uparrow}$ and $n_{i\downarrow}$, decrease differently. As a result, the spin polarization emerges in each dot. (2) When the level ε matches with the chemical potential

μ_L or μ_R, both Δn_1 and Δn_2 arrive at the maximum values. (3) When $\varepsilon = 0$, the spin accumulation in QDi is identically zero, i.e. $\Delta n_1 = \Delta n_2 = 0$. (4) QD1 and QD2 always accumulate an equal number of electrons and the directions of the spin polarization of two dots always remain opposite to each other. Moreover, the direction and strength of the spin accumulation of each dot can be easily controlled by tuning the level ε.

Why can spin accumulation be generated in the present device? Consider an electron propagating from the QD1 to the left lead or vice versa; it has two paths: one path is the direct tunnelling, the other is to first travel to the right lead and follow up with a tunnelling to the QD2 and then tunnel to left lead. If only to consider the first-order tunnelling process, the total effective coupling strength $T_{L1\sigma}$ between QD1 and the left lead can be expressed as[8,14]

$$\begin{aligned} T_{L1\sigma} &= \left|t_{L1\sigma} + t_{R1\sigma}(-i\pi\rho)t^*_{R2\sigma}g^r_{22\sigma}t_{L2\sigma}\right|^2 \\ &= \left|t_{L1}\right|^2 + \left|\pi\rho t_{R1}t_{R2}g^r_{22\sigma}t_{L2}\right|^2 \\ &+ 2\pi\rho\left|t_{L1}t_{R1}t_{R2}g^r_{22\sigma}t_{L2}\right|\sin\sigma\varphi_R, \end{aligned} \tag{9}$$

and the effective coupling strength $T_{R1\sigma}$ between the QD1 and the right lead is

$$\begin{aligned} T_{R1\sigma} &= \left|t_{R1\sigma} + t_{L1\sigma}(-i\pi\rho)t^*_{L2\sigma}g^r_{22\sigma}t_{R2\sigma}\right|^2 \\ &= \left|t_{R1}\right|^2 + \left|\pi\rho t_{L1}t_{L2}g^r_{22}t_{R2}\right|^2 \\ &- 2\pi\rho\left|t_{R1}t_{L1}t_{L2}g^r_{22\sigma}t_{R2}\right|\sin\sigma\varphi_R. \end{aligned} \tag{10}$$

In the same way, we can obtain the spin-dependent coupling strengths between QD2 and the leads,

$$\begin{aligned} T_{L2\sigma} &= \left|t_{L2\sigma} + t_{R2\sigma}(-i\pi\rho)t^*_{R1\sigma}g^r_{11\sigma}t_{L1\sigma}\right|^2 \\ &= \left|t_{L2}\right|^2 + \left|\pi\rho t_{R1}t_{R2}g^r_{11\sigma}t_{L1}\right|^2 \\ &- 2\pi\rho\left|t_{L1}t_{R1}t_{R2}g^r_{11\sigma}t_{L2}\right|\sin\sigma\varphi_R, \end{aligned} \tag{11}$$

$$\begin{aligned} T_{R2\sigma} &= \left|t_{R2\sigma} + t_{L2\sigma}(-i\pi\rho)t^*_{L1\sigma}g^r_{11\sigma}t_{R1\sigma}\right|^2 \\ &= \left|t_{R2}\right|^2 + \left|\pi\rho t_{L1}t_{L2}g^r_{11}t_{R1}\right|^2 \\ &+ 2\pi\rho\left|t_{R1}t_{L1}t_{L2}g^r_{11\sigma}t_{R2}\right|\sin\sigma\varphi_R, \end{aligned} \tag{12}$$

where ρ is the density of state in the leads. Generally, the effective coupling strengths of different spin directions in Eqs. (9)–(12) are different in the presence of RSO interaction. If $T_{L1\uparrow} > T_{L1\downarrow}$ in the case of $\varepsilon_{1\sigma} = \varepsilon_{2\sigma} = \varepsilon$ and the coupling of the dots to both leads is symmetrical, $T_{R1\downarrow} > T_{R1\uparrow}$ and $T_{R2\downarrow} = T_{L2\uparrow} < T_{R2\uparrow} = T_{L2\downarrow}$. Then it is easier for spin up (down) electron to tunnel from the left lead into QD1 (2) than that for spin down (up) electron, but it is more difficult to tunnel out from QD1 (2) to the right lead. Therefore spin accumulation emerges in QD, and QD1 and QD2 will accumulate an equal number of electrons of opposite spin directions.

In Fig. 3, we show Δn_1 and Δn_2 dependences on the phase φ_R. When φ_R is switch on, the spin accumulation emerges in QD. The electron spin directions in the two dots remain opposite to each other, and the magnitude of the spin accumulation is sensitive to φ_R. Δn_1 and Δn_2 exhibit a periodic function with a period of 2π. When φ_R is tuned from $-\pi/2$ to $\pi/2$, the spin polarization Δn_1 strongly varies from the largest negative value to the largest positive value and the polarization Δn_2 strongly varies from the maximum positive value to the maximum negative value.

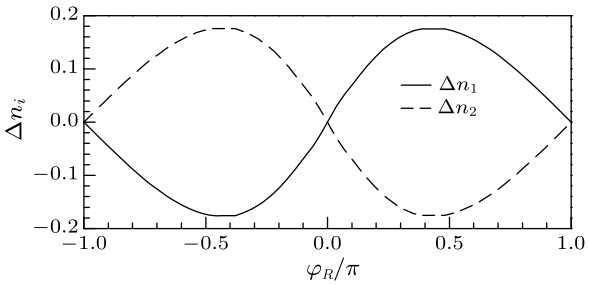

Fig. 3. Spin accumulation Δn_i vs φ_R with $V = 2$. The other parameters are $\varepsilon_1 = \varepsilon_2 = -1$, $K_BT = 0$ and $U = 0$.

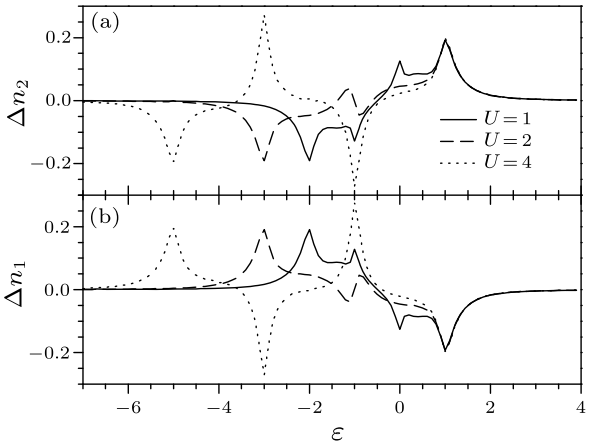

Fig. 4. Δn_i vs the intradot level ε with the bias $V = 2$, and $U = 1$ (solid), (dashed) and 4 (dotted). (a) Δn_1; (b) Δn_2. The other parameters are $\varphi_R = \pi/2$ and $K_BT = 0$.

In the following, we investigate the effect of the intradot Coulomb interaction. As shown in Figs. 4(a) and 4(b), the spin accumulations in two dots always remain opposite of each other even though the intradot Coulomb interaction is considered. It has the following feature: the maximum value of spin accumulation emerges as soon as the level ε or its Coulomb counterpart $\varepsilon + U$ equals the chemical potential of the lead (i.e.,μ_L and μ_R). For example, for the case of $U = 4$, there are four peaks in the plot of spin accumulation, corresponding to the levels $\varepsilon = -5, -3, -1$, 1, respectively. The same results can also be drawn from Fig. 5, which describes the relation of the spin accumulation to the bias with a fixed RSO coupling strength and different values of intradot Coulomb interaction. One can see that the maximum values of the spin accumulation locate at the bias $|V/2| = \varepsilon + U$, namely, the relation $\varepsilon + U = \mu_L$ or μ_R is satisfied. In addition, the direction of spin accumulation can be modulated by the intradot Coulomb interaction. For example, at the level $\varepsilon = -3$, Δn_1 is positive

with $U = 2$, while Δn_1 is negative with $U = 4$ [see Fig. 4(b)]. It is worth mentioning that when $\varepsilon = 0$ and $U = 0$, the spin accumulation never emerges for any values of the bias. The reason is that the level position is always located at the centre of the bias window, and has no chance to equal the chemical potential of the leads. In addition, compared with the case without intradot interaction, the spin accumulation in each dot is increased by the intradot Coulomb interaction U [see Fig. 5]. This can be explained from the plot of Coulomb interaction-dependent spin occupation numbers of different spin components in dot 1. As shown in Fig. 6, when the chemical potential μ_R passes the level $\varepsilon+U$, $n_{1\downarrow}$ strongly decreases, then $n_{1\uparrow}$ slightly decreases. Moreover, when $\mu_L = \varepsilon + U$, $n_{1\downarrow}$ and $n_{1\downarrow}$ increase differently. Consequently, two peaks of the spin accumulation emerge due to the repulsive Coulomb interaction. While $U = 0$, $n_{1\uparrow}$ and $n_{1\downarrow}$ always maintain the value of 0.5. As a result, the spin accumulation never emerges.

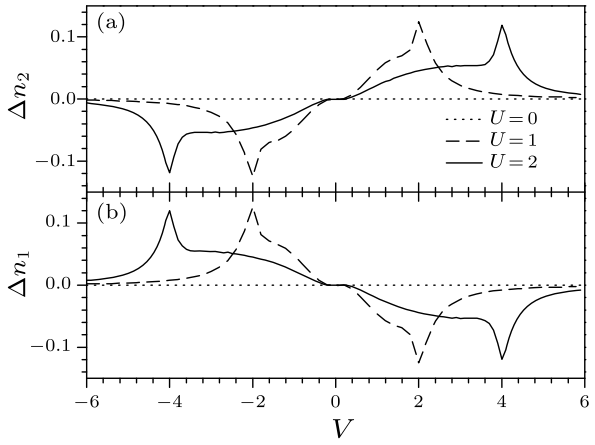

Fig. 5. Δn_i vs the bias V with the intradot level $\varepsilon = 0$, and $U = 0$ (dotted), 1 (dashed) and 2 (solid). (a) Δn_1; (b) Δn_2. The other parameters are the same as Fig. 4.

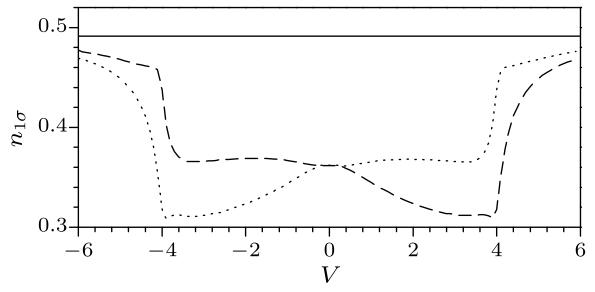

Fig. 6. Behaviour of $n_{1\sigma}$ vs bias V for different Coulomb interactions with $\varepsilon = 0$. The solid curves represents $n_{1\uparrow}$ and $n_{1\downarrow}$ for $U = 0$, the dotted curves (the dashed curves) are $n_{1\downarrow}(n_{1\uparrow})$ for $U = 2$. The other parameters are the same as Fig. 4.

We now briefly discuss the experimental realization of the present device. Firstly, the two-terminal DQD structure has already been realized in experiment.[15] As we discussed above, most of the spin accumulation control can be fulfilled by adjusting the bias voltage as well as the gate voltage. This indicates that the manipulation of the spin accumulation in this system is easy to realize in experiment. Secondly, the RSO interaction induced the phase factor $\varphi_{Ri} = \alpha L_i m^*/\hbar^2$ (where α is the RSO interaction strength, L_i is the length of dot i and m^* is the electron effective mass) is tunable in experiments. For some semiconductors, the RSO interaction strength α is about 3×10^{-11} eVm, which is reported in Refs. [16,17]. Then, if the dimension of QD is the typical value 100 nm and $m^* = 0.036m_e$, φ_{Ri} can reach a value of $\pi/2$ or larger in experiments.

In conclusion, we have investigated the spin accumulation of the double quantum dots embedded in an Aharonov–Bohm interferometer in term of the nonequilibrium Green's functions. The spin accumulation in QD emerges in the presence of RSO interaction even though no magnetic materials and magnetic field. It is found that the following objectives can be achieved: (1) The magnitude and direction of the spin accumulation in the QD can be controlled and tuned by the gate voltage or the bias. (2) The spin accumulation in two dot oscillates with a period of 2π as function of φ_R. (3) The position of the maximum of the spin accumulation can be modulated by the level, the intradot Coulomb interaction and the bias together. The tunable spin accumulation exhibited by the present system may have applications in the quantum computing.

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