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An Inequality Concerning Quantitative Duality in an Asymmetry Two-Way Interferometer *

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We reinvestigate the measures concerning the notion of duality of a quantum system (the "quanton") in an asymmetry two-way interferometer. A new measure of the which-way information is introduced, which is based on the fidelity $\mathcal{F}$ instead of the trace distance $\mathcal{Q}$ of the final states of the which-way detector. An improved inequality is derived to be $\mathcal{V}^2 \leq (\mathcal{R}_0^2 - \mathcal{P}^2)(1-\mathcal{F})^2$, which is more stringent than the previous one of $\mathcal{V}^2 \leq (1-\mathcal{P}^2)(1-\mathcal{Q}^2)$.

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The complementarity is one of the foundations of quantum mechanics.[1] It states that the quanton possesses properties are equally real but mutually exclusive. A well-known example is the wave-particle duality, which points out that the observation of an interference pattern and the acquisition of which-way information (WWI) are mutually exclusive in an interferometer. The situations "perfect fringe pattern with no WWI" and "full WWI with no fringes" are well-known. However, the issue of the intermediate stages have attracted great interest both theoretically[2−4] and experimentally[5,6] in the past decades.

Quantifying the two components of the wave-particle duality and deriving the mathematical relationship between them are the core tasks of this issue. The fringe pattern is standardly measured by the fringe visibility $\mathcal{V}$. However, how to quantify WWI is far from settled. In order to quantify WWI, Englert[4] introduced the distinguishability $\mathcal{D}$, i.e., the amount of the potential which-way information available to the experimenters. The relationship between $\mathcal{D}$ and $\mathcal{V}$ is shown by an inequality[4]

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1, \tag{1}$$

indicating that the partial WWI and partial fringe visibility are compatible. However, $\mathcal{D}$ includes two sources of WWI. One is the a priori WWI denoted by $\mathcal{P}$, i.e., a priori knowledge that the experimenter has about the way from the initial state of the quanton and the structure of the interferometer. The other one is due to a which-way detector (WWD), which is placed in the middle stage of the interferometer by the experimenter for acquiring extra WWI (see Fig. 1). In order to isolate the contribution to the distinguishability arising from the detector, a quantity $\mathcal{Q}$ was introduced in Ref. [7], which describes the ability for acquiring WWI of a WWD. Instead of Eq. (4), an inequality has been derived in Ref. [7],

$$\mathcal{V}^2 \leq (1-\mathcal{P}^2)(1-\mathcal{Q}^2). \tag{2}$$

In Ref. [7], $\mathcal{Q}$ is defined as the distance between the final states of the WWD corresponding to the two ways, in the norm of trace distance. Since $\mathcal{Q}$ is not related to the initial state of the quanton, Eq. (2) is successful in separating the two sources of WWI.

In this Letter, we introduce a new quantity to measure how good a WWD is, the "quality factor" $\mathcal{F}$. $\mathcal{F}$ is also defined by the distance between the two final states of the WWD corresponding to the two ways, in the norm of fidelity. Based on $\mathcal{F}$, some quantities are redefined and then an inequality is derived. This inequality is proved to be more stringent than Eq. (2) and a clear physical significance associated to the theory of unambiguous discrimination (UD) of quantum states is given in the text. What is more, the connection between $\mathcal{D}$ and the minimum-error discrimination (MD) of the quantum state has been investigated. We show that the position of $\mathcal{D}$ in Eq. (1) can be replaced by its counterpart in the theory of UD, which is also expressed by the notion of fidelity.

In an asymmetry two-way interferometer depicted in Fig. 1, the quanton is initially prepared in the state

$$\begin{aligned}\rho_Q^{(0)} &= \frac{1}{2}\left(1 + \boldsymbol{s}_Q^{(0)} \cdot \boldsymbol{\sigma}\right) \\ &= \frac{1}{2}\left(1 + s_{Qx}^{(0)}\sigma_x + s_{Qy}^{(0)}\sigma_y + s_{Qz}^{(0)}\sigma_z\right),\end{aligned} \tag{3}$$

where $\sigma = \left(\sigma_x, \sigma_y, \sigma_z\right)$ are Pauli's spin operator and $\boldsymbol{s}_Q^{(0)}$ is the initial Bloch vector of the quanton. Let $\mathcal{R}_0 = |\boldsymbol{s}_Q^{(0)}|$ represent the purity of $\rho_Q^{(0)}$, which is the length of the initial Bloch vector quantifies.

The phase shifter (PS) effects a transition through the operator

$$U_{\rm PS} = \exp\left(i\frac{\phi}{2}\sigma_z\right). \tag{4}$$

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The beam merger (BM) is characterized by the operator

$$U_{\rm BM} = \exp\left(i\frac{\pi}{4}\sigma_y\right). \quad (5)$$

Supposing that there is no WWD, the PS and BM turn the initial state $\rho_Q^{(0)}$ to the output state

$$\rho_Q^{(\rm out)} = U_{\rm BM}^\dagger U_{\rm PS}^\dagger \rho_Q^{(0)} U_{\rm PS} U_{\rm BM} = \frac{1}{2}(1 + \boldsymbol{s}_Q^{(\rm out)} \cdot \boldsymbol{\sigma}), \quad (6)$$

where

$$\boldsymbol{s}_Q^{(\rm out)} = \big(s_{Qz}^{(0)}, \sin\phi s_{Qx}^{(0)} + \cos\phi s_{Qy}^{(0)}, \sin\phi s_{Qy}^{(0)} - \cos\phi s_{Qx}^{(0)}\big). \quad (7)$$

The probability of measuring the upper or lower state of the quanton, after many repetitions of the experiments, displays an interference pattern versus variation of the phase in the PS. The fringe visibility is determined by the maximum and minimum values of the probability distribution with the formula

$$\mathcal{V} = \frac{p_{\max} - p_{\min}}{p_{\max} + p_{\min}}. \quad (8)$$

Hence the a priori fringe visibility (with no WWD) can be derived with Eqs. (6) and (8),

$$\mathcal{V}_0 = \big[(s_{Qx}^{(0)})^2 + (s_{Qy}^{(0)})^2\big]^{1/2}. \quad (9)$$

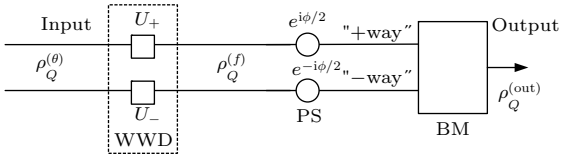

Fig. 1. Schematic two-way interferometer setup composed of a quantum which-way detector (WWD), a phase shifter (PS) and a beam merger (BM). The WWD is characterized by unitary operators $U_\pm$.

If we directly measure the observable σ_z on the $\rho_Q^{(0)}$, we could determine which of the two ways the quanton has taken on the average. The probabilities for taking the $\pm$ way are then given by

$$p_\pm = {\rm tr}\Big(\frac{1 \pm \sigma_z}{2}\rho_Q^{(0)}\Big) = \frac{1}{2}(1 \pm s_{Qz}^{(0)}), \quad (10)$$

respectively. The magnitude of their distance is the predictability of the ways of $\rho_Q^{(0)}$,

$$\mathcal{P} = |p_+ - p_-| = |s_{Qz}^{(0)}|. \quad (11)$$

In terms of WWI, the predictability $\mathcal{P}$ is a quantitative measure of the a prior knowledge that the experimenter has about the ways for $\rho_Q^{(0)}$. If one bets on the more probable way, then the probability of predicting the way correctly is $\frac{1}{2}(1 + \mathcal{P})$. In the Bloch sphere, $\mathcal{P}$ is the projection of $\boldsymbol{s}_Q^{(0)}$ in the z axis. We have

$$\mathcal{P}^2 + \mathcal{V}_0^2 = \mathcal{R}_0^2 \le 1. \quad (12)$$

As required by duality, Eq. (12) reveals the degradation of a priori fringe visibility $\mathcal{V}_0$ caused by $\mathcal{P}$, i.e., one source of WWI.

Consider the other source of WWI stemming from the quantum properties of the WWD. Suppose that the detector is prepared in the state $\rho_D^{(0)}$. As a result of the interaction between the quanton and the WWD, the evolution of the WWD can be characterized by

$$\rho_D^{(0)} \to U_\pm^\dagger \rho_D^{(0)} U_\pm = \rho_D^{(\pm)}, \text{ for the } \sigma_z = \pm 1 \text{ way}, \quad (13)$$

where U_+ and U_- are unitary operators acting exclusively on the detector. In order to separate the contribution to the total WWI by the two sources, we define the quality factor of the WWD as

$$\mathcal{F} = 1 - \mathbb{f} = 1 - {\rm tr}_D\sqrt{\rho_D^{(+)1/2}\rho_D^{(-)}\rho_D^{(+)1/2}}, \quad (14)$$

where $\mathbb{f}$ is the fidelity of states $\rho_D^{(-)}$ and $\rho_D^{(+)}$. The significance of $\mathcal{F}$ can be interpreted by the quantum state discrimination theory as follows. Suppose that the input of the interferometer is randomly prepared in state $|0\rangle$ (+way) or $|1\rangle$ (−way) with equal probability, the final state of the WWD will evolve to $\rho_D^{(+)}$ or $\rho_D^{(-)}$ correspondingly. An experimenter can get to know which way the quanton has taken via unambiguously discriminating the final state of the WWD. Based on the theory of quantum state discrimination,[9] $\mathcal{F}$ is the optimal successful probability of unambiguous discrimination (UD) of the two states $\rho_D^{(+)}$ and $\rho_D^{(-)}$. Hence, the quality factor $\mathcal{F}$ quantitatively measures the WWD's intrinsic ability to distinguish the ways. Its calculated value can be experimentally checked via the procedure of UD described above or along the lines described in Ref. [7] for measuring $\mathcal{Q}$. It should be noted that if both $\rho_D^{(+)}$ and $\rho_D^{(-)}$ have full rank, $\mathcal{F}$ can not be directly regarded as an optimal successful probability of UD because UD is now impossible. However, even in this extreme situation, the quantity $\mathcal{F}$, which directs the distance between $\rho_D^{(+)}$ and $\rho_D^{(-)}$, cna still be a good measure of the distinguishability of the two final states.

Let $\mathcal{W}_D$ denote how much WWI has been stored in the WWD in the interferometer. A basic idea of this study is that $\mathcal{W}_D$ depends not only on the quality factor $\mathcal{F}$ of WWD but also on the initial state of the quanton. This idea is based on the following observation. As demanded by duality, the emergence of $\mathcal{W}_D$ should lead to degradation of $\mathcal{V}$. However, if the a priori fringe visibility $\mathcal{V}_0 = 0$ (for instance, $\rho_Q^{(0)} = |0\rangle\langle 0|$ with $\mathcal{P} = 1$), $\mathcal{V}$ can not decrease anymore so that $\mathcal{W}_D = 0$ no matter what value $\mathcal{F}$ is. Hence we define $\mathcal{W}_D$ as the product of a priori fringe visibility of the input quanton $\mathcal{V}_0$ and $\mathcal{F}$

$$\mathcal{W}_D = \mathcal{V}_0\mathcal{F}. \quad (15)$$

Let us consider the situation that the detector begins with pure state $|\psi_D\rangle$. We aim to find the relationships among the quantities defined in the previous section. The unitary operator introduced by the WWD can be written as

$$U_C = \frac{1+\sigma_z}{2}U_+ + \frac{1-\sigma_z}{2}U_-. \tag{16}$$

After interacting with the WWD, the state of the quanton evolves to

$$\begin{aligned}\rho_Q^{(0)} \to \rho_Q^{(f)} &= \mathrm{tr}_D\big(U_c[\rho_Q^{(0)} \otimes |\psi_D\rangle\langle\psi_D|]U_c^\dagger\big) \\ &= \frac{1}{2}(1 + \boldsymbol{s}_Q^{(f)} \cdot \boldsymbol{\sigma}),\end{aligned} \tag{17}$$

with

$$\begin{aligned}\boldsymbol{s}_Q^{(f)} = \big(&\mathbb{f}\, s_{Qx}^{(0)}\cos\gamma + \mathbb{f}\, s_{Qy}^{(0)}\sin\gamma, \\ &\mathbb{f}\, s_{Qx}^{(0)}\sin\gamma - \mathbb{f}\, s_{Qy}^{(0)}\cos\gamma, s_z\big),\end{aligned} \tag{18}$$

where $\langle\psi_D|U_-^\dagger U_+|\psi_D\rangle = \mathbb{f}\, e^{i\gamma}$. Following Eq. (12), the fringe visibility then is given by

$$\mathcal{V} = \big[(s_{Qx}^{(f)})^2 + (s_{Qy}^{(f)})^2\big]^{1/2} = \mathcal{V}_0\mathbb{f}. \tag{19}$$

Combining Eq. (19) with Eq. (15) and using Eqs. (12) and (14), we have

$$\mathcal{V} + \mathcal{W}_D = \mathcal{V}_0 = (\mathcal{R}_0^2 - \mathcal{P}^2)^{1/2}. \tag{20}$$

Equation (20) reveals the relationship among the quantities $\mathcal{R}_0$, $\mathcal{P}$, $\mathcal{V}$ and $\mathcal{W}_D$. From Eq. (20) one can explicitly see that $\mathcal{W}_D$ quantifies the lost of the fringe visibility. To establish a correspondence between Eq. (2) and Eq. (20), we insert Eq. (15) into Eq. (20) and transform the equality to

$$\mathcal{V}^2 = (\mathcal{R}_0^2 - \mathcal{P}^2)(1-\mathcal{F})^2. \tag{21}$$

In the case that the WWD is allowed to be prepared in a mixed state $\rho_D^{(0)}$, Eq. (20) will turn to an inequality. This can be explained via the purification of $\rho_D^{(0)}$. By introducing an ancilla system A, we define a pure state $|\psi\rangle_{AD}$ for the joint system of A and WWD such that $\rho_D^{(0)} = \mathrm{tr}_A(|\psi\rangle_{AD}\langle\psi|)$. The final states of the joint system corresponding to the two ways are $|\psi^+\rangle_{AD} = U_+ \otimes I|\psi\rangle_{AD}$ and $|\psi^-\rangle_{AD} = U_- \otimes I|\psi\rangle_{AD}$. Following Eqs. (14) and (15), the WWI stored in the joint system is $\mathcal{W}_{AD} = \mathcal{V}_0(1 - |_{AD}\langle\psi^+|\psi^-\rangle_{AD}|)$. Equation (20) reduces to

$$\mathcal{V} + \mathcal{W}_{AD} = \mathcal{V}_0 = (\mathcal{R}_0^2 - \mathcal{P}^2)^{1/2}. \tag{22}$$

Since the WWI in the system A (usually is the environment) is not acquirable, the WWI stored in WWD must be less than $\mathcal{W}_{AD}$, i.e., $\mathcal{W}_D \le \mathcal{W}_{AD}$. This can be proved mathematically. The proof is based on Uhlmann's theorem,[10] which reads

$$\mathbb{f}\big(\rho_D^{(+)}, \rho_D^{(-)}\big) = \mathrm{MAX}|_{AD}\langle\psi^+|\psi^-\rangle_{AD}|, \tag{23}$$

where $\rho_D^{(+)}$ and $\rho_D^{(-)}$ denote the final states of the WWD corresponding to the two ways and the maximization is over all purifications $|\psi^+\rangle_{AD}$ of $\rho_D^{(+)}$ and $|\psi^-\rangle_{AD}$ of $\rho_D^{(-)}$. We have

$$\begin{aligned}\mathcal{W}_D &= \mathcal{V}_0\big(1 - \mathbb{f}(\rho_D^{(+)}, \rho_D^{(-)})\big) \\ &\le \mathcal{V}_0(1 - |_{AD}\langle\psi^+|\psi^-\rangle_{AD}|) = \mathcal{W}_{AD}.\end{aligned} \tag{24}$$

Accordingly, it is easy to obtain the transformed expressions of Eqs. (20) and (21) in the case that the WWD begins with the mixed state,

$$\mathcal{V} + \mathcal{W}_D \le \mathcal{V}_0 = (\mathcal{R}_0^2 - \mathcal{P}^2)^{1/2}, \tag{25}$$

$$\mathcal{V}^2 \le (\mathcal{R}_0^2 - \mathcal{P}^2)(1-\mathcal{F})^2, \tag{26}$$

where the equal sign holds if the WWD is prepared in a pure state. Eq. (26), which gives a quantitative description about duality, is the central result of this study.

Let us compare Eq. (26) with Eq. (2). In Eq. (2) the quality $\mathcal{Q}$ denotes the distance between $\rho_D^{(+)}$ and $\rho_D^{(-)}$ in the norm of trace distance

$$\mathcal{Q} = \frac{1}{2}\mathrm{tr}_D|\rho_D^{(+)} - \rho_D^{(-)}|. \tag{27}$$

The relationship between the two kinds of distance measure has been derived as[11]

$$(1-\mathcal{F})^2 = \mathbb{f}^2 \le 1 - \mathcal{Q}^2, \tag{28}$$

Noting Eq. (12) and Eq. (28), we can easily derive Eq. (2) from Eq. (26) under the replacement

$$(1-\mathcal{F})^2 \to 1 - \mathcal{Q}^2, \quad \mathcal{R}_0^2 \to 1. \tag{29}$$

The derivation shows that Eq. (26) is more stringent than Eq. (2). Another remarkable difference between Eq. (26) and Eqs. (1) and (2) is shown by introducing $\mathcal{R}_0$, the purity of the quanton, into the inequality. $\mathcal{R}_0$ allows one to accurately measure the effect of the purity of the input quanton on the final fringe visibility. An extreme example is given below to illustrate the necessity of introducing $\mathcal{R}_0$ into the inequality. Assume $\rho_Q^{(0)}$ is the most mixed state and $\rho_D^{(+)} = \rho_D^{(-)}$. From Eq. (26), one can calculate that $\mathcal{P} = \mathcal{Q} = \mathcal{F} = \mathcal{D} = 0$. Compared with Eqs. (1) and (2) which give the trivial inequality $\mathcal{V} \le 1$, our inequality shows the more stringent answer $\mathcal{V} = 0$. In fact, the fringe visibility $\mathcal{V}$ and the two sources of the WWI ($\mathcal{P}$ and $\mathcal{W}_D$) are bounded by the amount of purity $\mathcal{R}_0$ of the input quanton under the interplay relationship described by Eqs. (25) and (26). As in the example above, $\mathcal{R}_0 = 0$ directly determines that $\mathcal{V} = \mathcal{P} = \mathcal{W}_D = 0$, no matter what values $\mathcal{F}$ and $\mathcal{Q}$ are. Additionally, it should be noted that $\mathcal{R}_0$ is a experimentally measurable quantity. Comparing $\mathcal{R}_0 = \big(|s_{Qx}^{(0)}|^2 + |s_{Qy}^{(0)}|^2 + |s_{Qz}^{(0)}|^2\big)^{1/2}$ with $\mathcal{P} = |s_{Qx}^{(0)}|$, the calculated value of $\mathcal{R}_0$ can be

checked by repeating experiments in a similar way to check the value of $\mathcal{P}$.

From Eq. (26), we can see that $\mathcal{F}$ is a good measure of the WWD's intrinsic ability of acquiring WWI from the quanton. If $\mathcal{F}=0$, the two final states of WWD corresponding to the two ways can not be unambiguously discriminated so that no WWI is extracted and $\mathcal{V}$ does not change. If $\mathcal{F}=1$, the two final states can be unambiguously discriminated with probability 1. The WWD gets full ability and acquires WWI as much as the quanton has. Hence no matter what value $\mathcal{P}$ and $\mathcal{R}_{\prime}$ are, the fringe visibility $\mathcal{V}$ vanishes.

We have already generalized Eq. (2) to Eq. (26) associated by the theory of UD. It is interesting to ask what will happen if we connect Eq. (1) with the theory of UD. In fact, the distinguishability $\mathcal{D}$ in Eq. (1), which reads

$$\mathcal{D}=\mathrm{tr}_D\left|p_+\rho_D^{(+)}-p_-\rho_D^{(-)}\right|, \tag{30}$$

is naturally relative to the minimum-error discrimination (MD), i.e., an ambiguous discrimination with a minimum probability of inferring an erroneous result. The optimal maximum-correct probability of MD between $\rho_D^{(+)}$ and $\rho_D^{(-)}$ with the rising probability p_+ and p_- can be expressed by[12]

$$\mathcal{L}=\frac{1}{2}(1+\mathcal{D}), \tag{31}$$

which is named as the optimal likelihood for guessing the right way in Ref. [4]. Therefore $\mathcal{D}$ quantitatively measures the distinguishability of $\rho_D^{(+)}$ and $\rho_D^{(-)}$ with the rising probability p_+ and p_- in the sense of MD. The counterpart of $\mathcal{D}$ in the theory of UD is $\mathcal{S}$, the successful probability of UD between $\rho_D^{(+)}$ and $\rho_D^{(-)}$ with the prior probabilities p_+ and p_-. Without losing generality, let $p_+\geq p_-$, and $\mathcal{S}$ obeys the inequality,[9]

$$\begin{aligned}&\mathcal{S}\leq 1-2\sqrt{p_+p_-}\mathbb{f}, \quad \frac{1}{2}\leq p_+\leq\frac{1}{1+\mathbb{f}^2},\\ &\mathcal{S}\leq 1-p_--p_+\mathbb{f}^2, \quad p_+\geq\frac{1}{1+\mathbb{f}^2}.\end{aligned} \tag{32}$$

Since $p_--p_+\mathbb{f}^2\geq 1-2\sqrt{p_+p_-}\mathbb{f}$, it is clear that the first line of Eq. (35) denotes the overall upper bound $\mathcal{S}_{up}$ on the successful probability

$$\mathcal{S}\leq\mathcal{S}_{\mathrm{up}}\equiv 1-2\sqrt{p_+p_-}\mathbb{f}, \tag{33}$$

for arbitrary values of the prior probabilities. In conjunction with Eq. (11), noting $\sqrt{p_+p_-}\leq 1/2$ and $\mathbb{f}\leq 1$ we have

$$\begin{aligned}&\frac{(\mathcal{R}_0^2-\mathcal{P}^2)(1-\mathcal{F})^2}{1-\mathcal{S}_{\mathrm{up}}^2}\\ &=\frac{(\mathcal{R}_0-p_++p_-)(\mathcal{R}_0+p_+-p_-)\mathbb{f}^2}{(4-4\sqrt{p_+p_-}\mathbb{f})\sqrt{p_+p_-}\mathbb{f}}\\ &\leq\frac{(1-p_++p_-)(1+p_+-p_-)\mathbb{f}^2}{(4-4\sqrt{p_+p_-}\mathbb{f})\sqrt{p_+p_-}\mathbb{f}}\\ &=\frac{\sqrt{p_+p_-}\mathbb{f}}{1-\sqrt{p_+p_-}\mathbb{f}}\leq 1,\end{aligned} \tag{34}$$

Inserting Eq. (37) into Eq. (26), one easily obtains the relationship

$$\mathcal{S}^2+\mathcal{V}^2\leq\mathcal{S}_{\mathrm{up}}^2+\mathcal{V}^2\leq 1, \tag{35}$$

which is in the same form as Eq. (1). Hence $\mathcal{S}$ can be regarded as an alternative definition of the distinguishability. The significance of $\mathcal{S}$, i.e., the optimal successful probability of UD between $\rho_D^{(+)}$ and $\rho_D^{(-)}$ with the probability p_+ and p_-, directly reveals the meaning of distinguishability as well as $\mathcal{D}$.

In conclusion, based on the notion of fidelity instead of trace distance, we redefine a series of quantities concerning the duality in an asymmetry interferometer. Then an improved inequality (Eq. (26)) related to these quantities is derived. Since the inequality Eq. (2)in the previous work can be a direct deduction of Eq. (26), Eq. (26) shows a more stringent relationship between the fringe visibility and the WWI. Moreover, we first reveal the association between the quantities concerning the duality and the theory of quantum state discrimination, which provides a new viewpoint for researching the duality quantitatively.

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