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Chaotic Control of Network Traffic *

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A method of chaotic control on network traffic is presented. By this method, the chaotic network traffic can be controlled to a pre-assigned equilibrium point according to chaotic prediction and the largest Lyapunov exponent of the traffic on congested link is reduced, thereby the probability of traffic burst and network congestion can be reduced. Numerical examples show that this method is effective.

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As an open system far away from the equilibrium state, in fact Internet is a dissipative system. The Internet traffic system has diverse features of nonlinear dynamics, and some researches[1,2] show that this system has low-dimensional chaotic nature.

Chaos is a phenomenon, by which low-order nonlinear dynamical systems exhibit complex, seemingly random behavior. A system is chaotic if it satisfies some conditions, including nonlinearity, order in disorder, butterfly effect, etc. We can observe the evolution of the chaotic system not only by a function of time, but also by drawing the trajectory of the system as it moves in a multi-dimensional state space, i.e., phase space. In phase space, each dimension represents a system variable and each point is a unique state of the system. After a long enough time, the trajectory becomes orderly, which is called the chaotic attractor. A chaotic attractor is a set of states in a system's phase space with an elegant and fixed geometric structure, but after a stretch-and-fold operation, it can be transformed into a time series, which is disorder and fractals. Hence research on the chaotic nature of network traffic is closely related to research on the self-similarity of network traffic.[3] In essence, nonlinear dynamics of a chaotic system brings a fractal space-time structure which displays self-similarity.[4]

The largest Lyapunov exponent (LLE)[5] is a common macro nonlinear characteristic quantity of chaotic systems, which quantifies the average exponential rates of divergence of nearby orbits in phase space. To estimate LLE, first the phase space should be reconstructed to unveil the dynamics in a high dimensional space. Phase space reconstruction[6] means that the output of the system (usually a time series) should be embedded into a higher dimensional state space. The phase space reconstruction needs two important parameters, i.e., time delay[7] and embedding dimension.[8] Then we can estimate the LLE using some common methods, such as wolf's method[9] or small data sets method.[10,11] In a chaotic network traffic system, a larger LLE implies that traffic burst occurs more frequently.

The fractal and chaotic characteristics of network traffic will lead to uncertain and unexpected behaviors, which greatly reduce the ability to control the network and increase of the probability of network congestion, but most of the traditional traffic control methods ignore the chaotic characteristics of network traffic. In this Letter we propose a chaotic control method for network traffic. By this method, the chaotic network traffic on the congested link can be controlled to a pre-assigned equilibrium point according to chaotic prediction, and the LLE of the traffic on the congested link is reduced, thereby the probability of traffic burst and network congestion decreases.

As the nonlinear dynamics of the network traffic system is unknown, this method is based on chaotic prediction. Here we choose the most commonly used local linear approximation (LLA) method[12] to predict the traffic. The LLA method is simple but has high prediction abilities, which makes use of the nearest neighbors in phase space. Given data $\{x_n\}_{n=1}^{N}$, to predict the next step of point x_k, K nearest neighbors to this point (i.e., $x_{\alpha_1}, x_{\alpha_2}, \ldots, x_{\alpha_K}$) are selected from the past data. The predicted future point x_{k+1} is determined by averaging of those selected neighbors

$$x_{k+1} = S(x_k) \approx \frac{1}{K}\sum_{k=1}^{K} x_{\alpha_k} \approx f(x_k), \qquad (1)$$

Then we present this chaotic control method. Consider a nonlinear discrete-time dynamical system of the form

$$x_{k+1} = f(x_k), \qquad x_k \in R^n, \qquad (2)$$

where the function f is unknown. Assume that f is

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continuously differentiable and any Jacobian of system (2) evaluated at x_k, denoted as $f'(x_k)$, is bounded.

Design a control input sequence u_k, then the output of the controlled system is

$$x_{k+1} = f(x_k) + u_k, \tag{3}$$

$$\begin{aligned} u_k &= Bx_k + C_k, \quad B = -f'(x^*) + E, \\ C_k &= a(1 - e^{\lambda(S(x_k)-x*)})(S(x_k) - x^*) + \varepsilon, \end{aligned} \tag{4}$$

where x^* is the fixed point which we want to guide the system to, $f'(x^*)$ is the Jacobian of system (2) evaluated at point x^*, $S(x_k)$ is the prediction of x_{k+1} using the LLA method, and E, a, λ, ε are the predefined constant.

Denoted $J(x_k)$ as the Jacobian of the system (3) evaluated at point x_k, hence $J(x^*)$ is given by

$$\begin{aligned} J(x^*) &= \frac{\partial x_{k+1}}{\partial x_k}\Big|_{x*} \\ &= E + aS'(x^*)\big(1 - e^{\lambda(S(x^*)-x^*)}\big) \\ &\quad \cdot (1 + S(x^*) - x^*), \end{aligned} \tag{5}$$

where $S'(x^*)$ is the Jacobian of the prediction of system (2), evaluated at point x^*, and $A = aS'(x^*)(1 - e^{\lambda(S(x^*)-x^*)})(1+S(x^*)-x^*)$ is a constant, we can tune E carefully to make sure that

$$|J(x^*)| = E + A < 1. \tag{6}$$

Then choose an appropriate value for ε to let x^* become a fixed point of system (3), that is,

$$\begin{aligned} x^* &= f(x^*) + Bx^* + a\big[1 - e^{\lambda(S(x^*)-x^*)}\big] \\ &\quad \cdot (S(x^*) - x^*) + \varepsilon, \end{aligned} \tag{7}$$

$|J(x^*)| < 1$ and f is continuously differentiable in an open neighbourhood of the fixed point x^*. Thus fixed point x^* is stable, which means that system (3) will converge to x^* after a finite number of iterations.

We can use a local linearization method[13] to estimate the Jacobian of an unknown dynamic system at x_k. Take the local region B of interest, where the point x_k locates. First consider a set $\{z(k,0)\}$ of differences, where $\{z(k,0)\} = \{x_k - \langle x_k \rangle\}$, $\langle x_k \rangle$ is the average position of all the points inside B. After a lag T, all the points in box B evolve by the lag T. Therefore, the set $\{z(k,0)\}$ could be mapped into $\{z(k,T)\} = \{x_{k+T} - \langle x_{k+T} \rangle\}$, where $\langle x_{k+T} \rangle$ is the average position of all the points inside B evolved after the evolution lag T. If the region B is small enough, the spatial evolution from $\{z(k,0)\}$ to $\{z(k,T)\}$ can be represented by a Jacobian matrix $J(x_k)$ given by

$$z(k,T) \approx J(x_k)z(k,0). \tag{8}$$

Equation (8) has a solution if the number of points in B is larger than the number of dimensions of x_k and $J(x_k)$ is nondegenerated. Consequently, we can calculate $J(x_k)$ using the least squares method. Both the approximation of $f'(x^*)$ and $S'(x^*)$ can be estimated by this method.

Then we show an example of the chaotic control to the chaotic traffic system of an actual network, to verify the proposed method.

As an illustration, consider the traffic data from the Abilene network (a high-performance network which serves Internet2).[14] A map of this network is shown in Fig. 1.

Fig. 1. Map of Abilene.

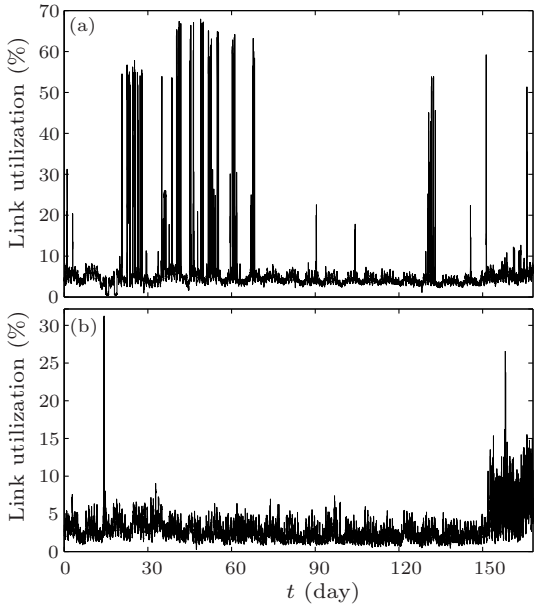

Fig. 2. Traffic data of uncontrolled system: (a) traffic data on link Indianapolis→Kansas City, (b) traffic data on link Indianapolis→Atlanta.

As an elephant flow from Chicago to Los Angeles, the path Chicago → Indianapolis → Kansas City → Denver → Sunnyvale → Los Angeles always experiences high traffic load, and the traffic data of link Indianapolis → Kansas City along this path is shown in Fig. 2(a). Meanwhile, the traffic load of the other optional path Chicago → Indianapolis → Atlanta → Houston → Los Angeles maintains a relatively low level, and the traffic data of link Indianapolis → Atlanta along this path is shown in Fig. 2(b).

We tune part of the traffic on the congested link Indianapolis → Kansas City to backup link Indianapolis → Atlanta and the details of chaotic control are as follows. Note that the traffic data corresponds to x_k in system (3).

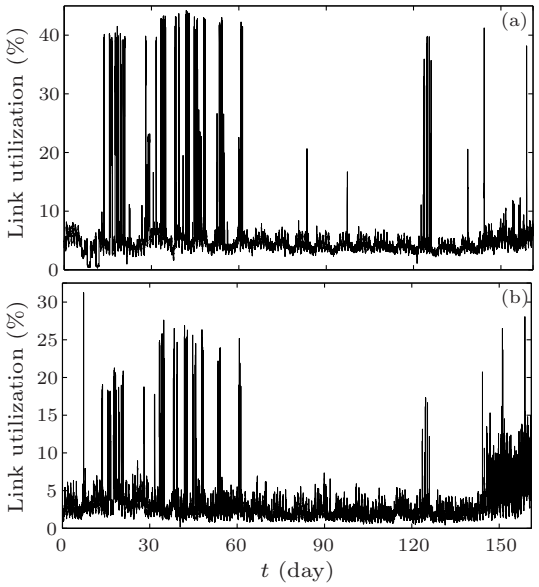

Fig. 3. The control results of traffic: (a) traffic data on link Indianapolis→Kansas City, (b) traffic data on link Indianapolis→Atlanta.

Consider the traffic data on link Indianapolis → Kansas City. We can see that there is no congestion from 1 March 2004 to 7 March 2004, and we start our chaotic control from 8 March 2004. The subsequent traffic data could be predicted by the LLA method. Based on actual demand, different control goals can be selected, and in this study we select the median of the traffic data from 1 March 2004 to 7 March 2004 as our control goal. Let $a = 1$, $\lambda = 0.005$, then $A = 0.0288$. Assign $E = 0.3$, hence $J(x^*) = E + A = e^{-1.1123}$ and $\varepsilon = 0.2297$, the design of the feedback controller in system (3) is completed.

We obtain the control results as shown in Fig. 3. The LLE of the controlled system can be calculated by the common wolf's method:[9] the LLE of traffic data on link Indianapolis → Kansas City decreased from 0.0297 to 0.0209, and the LLE of traffic data on link Indianapolis → Atlanta increased from 0.0169 to 0.0172. Overall, on link Indianapolis → Kansas City the probability of traffic burst and congestion will be greatly reduced, and on link Indianapolis → Kansas City the probability of traffic burst will improve slightly.

In conclusion, we propose a chaotic control method for network traffic. The largest Lyapunov exponent of chaotic network traffic could be reduced, and the probability of traffic burst and network congestion decreases. The numerical examples demonstrate its effectiveness.

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