CHIN. PHYS. LETT. Vol. 26, No. 12 (2009) 120301

Dynamics of Bright Solitons in Bose–Einstein Condensates with Complicated Potential *

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(Received 12 June 2009)

We present analytical solutions of the one-dimensional nonlinear Schrodinger equations of Bose–Einstein condensates in an expulsive parabolic background with a complex potential and gravitational field, by performing the Darboux transformation from a trivial seed solution. It is shown that under a safe range of parameter, the shape of bright soliton can be controlled well by adjusting the experimental parameter of the ratio of axial oscillation to radial oscillation and feeding condensates from a thermal cloud. The gravitational field can change the contrail of the bright soliton trains without changing their peak and width.

PACS: 03. 75. Lm, 02. 30. lk, 05. 45. Yv

There has been great interest in studying various properties of ultra cold atoms with the experimental realization of trapped Bose–Einstein condensates (BECs) in alkali atoms.[1,2] At very low temperatures, the dynamics of a BECs can usually be described by the nonlinear Schrodinger equation with a trap potential, namely the Gross–Pitaevskii (GP) equation. The phenomena governed by this nonlinear equation can be better understood with the help of exact solutions. For example, they can contribute to select the experimental parameter, to analyze the stability of BECs and to check numerical analysis of this nonlinear equation. Therefore, the construction of the exact solution of the GP equation is one of the most important and essential tasks. Thus lots of scientists devote themselves to solving the nonlinear equation exactly,[3−13] and in the past few decades, they have proposed many good methods to exactly solve the GP equation, such as the inverse scattering technique,[13] the symmetry group analysis method,[11] the mapping deformation method,[12] and the Hirota bilinear equation.[5] Liang *et al.* have solved the GP equation with parabolic potential by performing a Darboux transformation from a nontrivial seed solution.[3] However the identification of such nontrivial seed solutions has always been proved to be a major obstacle in the investigation of the nonlinear Schrodinger equation. In this Letter, we construct a series of nontrivial exact solutions from a trivial seed solution by the Darboux Transformation method.

Recently, there have been some experimental reports on solitons in the quasi one-dimensional (1D) system which were observed by magnetically tuning the atom-atom interaction from repulsive to attractive. Experimentally for the realization of 1D systems, one should make the radial frequency much larger than the axial frequency and strongly confine the radial motion. The state in this cylindrical harmonic-oscillator trap represents a nonlinear matter wave.[14] The formation and propagation of matter wave solitons are more interesting dynamical feature in Bose–Einstein condensation, such as dark solitons,[15−20] bright solitons,[21−26] and four-wave mixing.[27] For atoms in the nK–mK temperature regime, the effect of the Earth's gravitational field is by no means negligible especially in the case of magnetic trapping. The complex potential related to the feeding parameter from the thermal cloud also plays an important role in the evolution of solitons.

In this study, we study BECs in an expulsive parabolic background with a complex potential and gravitational field. It is shown that the gravitational field can change the contrail of the bright soliton trains without changing the peak and width of the soliton in the parabolic background. We infer that the peak and width of the bright soliton can be controlled well by adjusting the experiment parameter of the ratio of axial oscillation to radial oscillation and feeding condensates from thermal cloud. The evolution of soliton in linear potential is also studied. The dynamics character of soliton can be used to "atom laser" and the evolution of direction can be controlled by the gravitational field or the linear potential.

At the mean-field level, the Gross–Pitaevskii equation governs the evolution of the macroscopic wave function of Bose–Einstein condensates. The one-dimensional GP equation with an expulsive parabolic, linear and complex potential is

$$i\frac{\partial\psi}{\partial t}+\frac{\partial^2\psi}{\partial x^2}+a(t)|\psi|^2\psi+\frac{1}{4}\lambda^2x^2\psi+kx\psi+i\gamma\psi=0, \quad (1)$$

where the time t and coordinate x are measured in units $\frac{2}{\omega_\perp}$ and $a_\perp$, $a_\perp=\sqrt{\frac{\hbar}{m\omega_\perp}}$ and $a_0=\sqrt{\frac{\hbar}{m\omega_0}}$ are linear oscillator lengths in the transverse and cigar-axis directions respectively, $\omega_\perp$ is the radial oscillation frequency and ω_0 is the axial oscillation frequency, m is the atom mass, and $|\lambda|=2\frac{|\omega_0|}{\omega_\perp}\ll 1$, here the ax-

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ial of cylindrical harmonic trap is along the x direction which can be either confining or expulsive. γ is a small parameter related to the feeding parameter from the thermal cloud ($\gamma < 0$), $a(t) = 2g_0 e^{\lambda t + 2\gamma t} > 0$ is the scattering length dependent on the ratio of oscillation frequency and feeding parameter which can be adjusted by experiment, and $g_0 > 0$ is the scattering length at $t = 0$, kx responds to the gravitational field or some other linear potentials.

We assume the solution of Eq. (1) as follows:

$$\Psi = Q(x,t)\exp[-i\lambda x^2/4 - \lambda t/2 - \gamma t - ikx/\lambda]. \quad (2)$$

Substituting Eq. (2) into Eq. (1), we can obtain

$$iQ_t + Q_{xx} - \left[i\lambda x + \frac{i2k}{\lambda}\right]Q_x - i\left[\lambda - \frac{ik^2}{\lambda^2}\right]Q + 2g_0|Q|^2Q = 0. \quad (3)$$

Using a Lax-pair method and the Darboux transformation theory, we can obtain the exact solution of Eq. (1),

$$\psi = \frac{4\alpha_1 A_c \exp\theta}{\sqrt{g_0}[1 + A_c^2 \exp\varphi]}, \quad (4)$$

where

$$\theta = \frac{2i}{\lambda}(\alpha_1 - i\beta_1)^2 e^{2\lambda t} - \frac{4k}{\lambda^2}(\alpha_1 - i\beta_1)e^{\lambda t} - \frac{ik^2 t}{\lambda^2} - 2(\alpha_1 - i\beta_1)xe^{\lambda t} + \frac{\lambda t}{2} - \frac{i\lambda x^2}{4} - \gamma t - \frac{ikx}{\lambda},$$

$$\varphi = \frac{8\alpha_1\beta_1}{\lambda}e^{2\lambda t} - \frac{8k\alpha_1}{\lambda^2}e^{\lambda t} - 4\alpha_1 x e^{\lambda t},$$

where A_c, α_1, and β_1 are the arbitrary real constants. This solution describes the dynamics of bright solitons in BECs with time-dependent interaction in an expulsive parabolic background with linear potential and complex potential. It is a new solution which is different from the one provided by Li Biao *et al.* when $k = 0$,[9] and Liang *et al.* who have discussed the evolution of bright solitons when $\gamma = 0, k = 0$.[3]

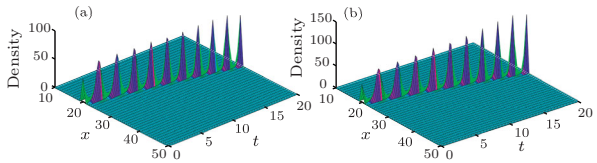


Fig. 1. Dynamics of bright solitons without gravitational field: (a) $\gamma = 0$, there is no gain atoms; (b)$\gamma = -0.01$, condensates gain atoms from thermal cloud. The other coefficients are $\alpha_1 = 2$; $\beta_1 = 0.2$; $\lambda = 0.02$; $A_c = 2$; $g_0 = 0.25$; $k = 0$.

If we ignore the effect of the gravitational field, that is $k = 0$, we can realize the dynamics of bright solitons in a parabolic background with or without complex potential, as shown in Fig. 1. From Fig. 1(a), we observe that the peak of matter wave density increases with the time evolution (the absolute value of the scattering length increases) while the number of condensate atoms is constant ($\gamma = 0$). Thus the matter wave density can be compressed into an assumed peak by adjusting the scattering length, which is dependent on the experimental parameter of the ratio of axial oscillation to radial oscillation (λ). Compared Fig. 1(b) with Fig. 1(a), it is seen that the peak of wave density in Fig. 1(b) increases more rapidly due to the feeding of cold atoms from the thermal cloud ($\gamma < 0$) with the same other parameters in Fig. 1(a). This means that feeding atoms from the thermal cloud can be helpful in increasing the peak of bright solitons. However, $a(t) = 2g_0 e^{\lambda t + 2\gamma t}$, the scattering length depends on both parameter λ and feeding parameter γ, the number of atoms in condensates changes with time for a given feeding parameter γ. In this case, the dynamics of soliton become slightly more complicated. In the following we mainly discuss the influence of the feeding parameter γ and λ on the dynamics of bright soliton in different conditions: (1) $\gamma > 0$, when $\lambda < 0$ the peak of soliton decreases and its width broadens with time, and the width changes more slowly when the value of λ approaches to zero; when $0 < \lambda < 2\gamma$, its peak and width both decrease with time evolution; when $\lambda = 2\gamma$, its peak is a constant $(4\alpha_1^2/g_o)$, as shown in Fig. 2, while its width is compressed; when $\lambda > 2\gamma$, the peak increases and its width is compressed; (2) $\gamma < 0$, when $\lambda < 2\gamma$ its peak decreases and its width broadens; when $\lambda = 2\gamma$ the peak is a constant $(4\alpha_1^2/g_o)$ and the width is broadened; when $2\gamma < \lambda < 0$, its peak and width both increase with time evolution; when $\lambda > 0$, the peak increases and its width compressed, while the width also changes more slowly when the value of λ approaches to zero.

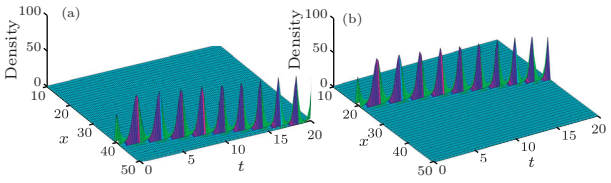


Fig. 2. (a) Dynamics of bright solitons with gravitational field, $k = -0.004$. (b) The dynamics of bright solitons without gravitational field, $k = 0$. The other coefficients are: $\alpha_1 = 2$; $\beta_1 = 0.2$; $\lambda = 0.02$; $A_c = 2$; $g_0 = 0.25$; $\gamma = 0.01$. The peaks evolve stably with time when $\lambda = 2\gamma$.

From the above discussion, we know that the dynamics of bright soliton can be determined by λ and γ. We find that the condition for a stable peak is $\lambda = 2\gamma$, and the peak value equals $4\alpha_1^2/g_0$. The width does not change with time evolution when $\lambda \to 0$. This provides us with a good way to control the bright soliton's shape. It should be noted that λ can not equal to zero in the solution (4). We will discuss the case that $\lambda = 0$ in the following.

To study the effects of the gravitational field on bright soliton in a parabolic background, we show the evolution of soliton with the gravitational field

($k = -0.004$) in Fig. 2(a), the bright soliton without the gravitational field is shown in Fig. 2(b) with the same other parameters. We observe that the gravitational field could not change the peak and width of matter wave, while the contrail of the bright soliton trains has an offset to $+x$ direction with reduction of the k value. By studying the sectional drawing of soliton at given times with different k, we find that the moving velocity of soliton along the x direction has no relations with linear potential. If there is no parabolic potential, what about the effects of linear potential on dynamics of bright soliton?

If there is no parabolic potential, the evolution of soliton should satisfy the equation

$$i\frac{\partial\psi}{\partial t} + \frac{\partial^2\psi}{\partial x^2} + a(t)|\psi|^2\psi + (kx + i\gamma)\psi = 0, \qquad (5)$$

where $a(t)$ becomes $2g_0e^{2\gamma t}$. This equation can be obtained from Eq. (1) by setting $\lambda = 0$, but the solution of Eq. (5) can not be reduced from Eq. (4) directly because λ cannot take on the values 0 in Eq. (4). Therefore we should resolve Eq. (5) exactly again by performing the Darboux transformation, its analytical solution can be given as follows:

$$\psi = \frac{4\alpha A_c}{\sqrt{g_0}}\frac{\exp\theta'}{1 + A_c^2\exp\varphi'}, \qquad (6)$$

where

$$\begin{aligned}\theta' =& -2(\alpha - i\beta)x + ikxt + 4i(\alpha - i\beta)^2t \\ &+ 2kt^2(\alpha - i\beta) - \frac{i}{3}k^2t^3 - \gamma t, \\ \varphi' =& -4\alpha x + 16\alpha\beta t + 4\alpha kt^2,\end{aligned}$$

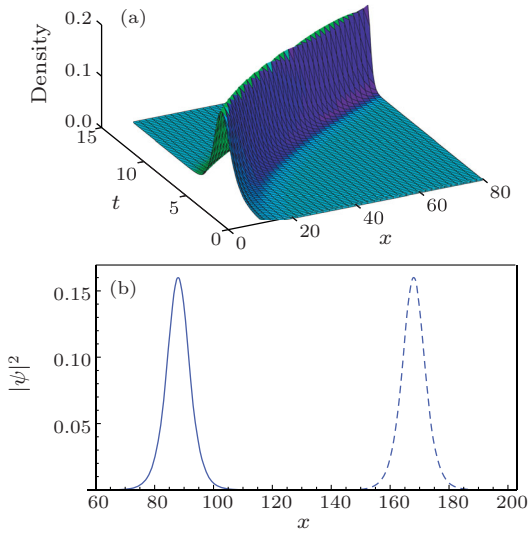


Fig. 3. (a) Evolution of soliton in a linear potentials $k = 0.4$; (b) sectional drawing of soliton at a given time ($t = 20$) with $k = 0.4$ (dashed line) and $k = 0.2$ (solid line); with the other coefficients $\alpha = 0.1$, $\beta = 0.1$, $g_0 = 0.25$, $A_c = 1$.

From the above solution, we know that the peak of soliton exponentially increases with time when the condensates gain atoms from the thermal cloud ($\gamma < 0$). Here we study the dynamics of the bright solitons in linear potential, the evolution of bright soliton is shown in Fig. 3(a). It is found that the peak is stable, and its width is compressed with time evolution. To study the effects of linear potential, we show a sectional drawing of soliton at a given time with different linear potentials in Fig. 3(b). We find that the shape of soliton is unchanged by varying the value of k with the same other coefficients, but its position is changed. At the given time for different k, it is seen that soliton moves along the $+x$ direction more quickly when the value of k increases. This effect is different from the effect of linear potential on soliton in parabolic background. Thus we can control the movement of soliton by linear potential, this character can be used to atom laser.

In summary, we have solved the 1D Gross–Pitaevskii (GP) equation analytically which describes BECs in an expulsive parabolic and complex potential considering the effects of the gravitational field or some other linear potentials. It is shown that under a safe range of parameter, there is an explicit way to control the soliton's shape by adjusting the experimental parameter of the ratio of axial oscillation to radial oscillation and feeding condensates from a thermal cloud. The gravitational field changes the contrail of the bright soliton trains without changing the peak and width of the soliton in the parabolic background. We find a good way to control the soliton's movement through linear potential without changing its shape, which can be used to the atom laser.

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