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Large Positive and Negative Lateral Shifts from an Anisotropic Metamaterial Slab Backed by a Metal *

CHENG Min(程敏)[1,2**], CHEN Rong(陈荣)[2]

[1]*Department of Physics and Electronic Information Engineering, Minjiang University, Fuzhou 350108*
[2]*Key Laboratory of Optoelectronic Science and Technology for Medicine (Ministry of Education), Fujian Normal University, Fuzhou 350007*

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The lateral shift of a light beam at the surface of an anisotropic metamaterial (AMM) slab backed by a metal is investigated. Analytical expressions of the lateral shifts are derived using the stationary-phase method, in the case that total reflection does and does not occur at the first interface. The sign of the lateral shift in two situations is discussed, and the necessary conditions for the lateral shift to be positive or negative are given. It is shown that the thickness and physical parameters of the AMM slab and the incident angle of the light beam strongly affect the properties of the lateral shift. Numerical results validate these conclusions. The lossy effect of the metamaterial on the lateral shift is also investigated.

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In some situations, a reflected beam may experience a lateral shift with respect to the path predicted by geometric optics. This lateral shift is referred to as the Goos–Hänchen effect[1] and was explained theoretically in terms of a stationary phase method.[2] Investigation of the Goos–Hänchen shift has been extended to multilayered structures containing various materials,[3−11] the nonlinear Goos–Hänchen shift[12−14] and the manipulation of the Goos–Hänchen shifts of light beam[15] and ballistic electron beam.[16]

Recently an artificial material with both negative permittivity and permeability, which is called the double negative metamaterial (DNM), has attracted a great deal of attention[17−22] due to its unusual electromagnetic properties, such as a negative refractive index[19] and ability to form a 'perfect lens'.[20] Recently, the Goos–Hänchen shift associated with DNM is of interest for potential device applications,[23,24] and the lateral shift of a optical beam from an interface separating DPM (double positive medium) and DNM has been extensively studied.[25−31] However, most of the artificial structures are actually anisotropic with respect to the direction of incident waves. Recently, the transmitted beam shifting for an anisotropic metamaterial slab has been discussed.[32−35] However, in Refs. [32–35] the amplitude of the beam is strongly dependent on the parameters of the slab, and little work has been carried out on the lateral shift of the reflected beam from a grounded anisotropic metamaterial slab (the lateral shifts of the reflected beam from a dielectric slab and an isotropic left-handed metamaterial slab have been discussed in Ref. [36,37]).

In this Letter, we consider a light beam reflecting from an anisotropic metamaterial slab backed by a metal, which can be considered as a perfect conductor that makes the magnitude of the reflection coefficient is almost equal to unity at any incident angles. The analytical expression for the lateral shift was obtained by the stationary-phase method, and we find that the properties of the lateral shift depend on the thickness and physical parameters of the anisotropic metamaterial slab and the incident angle. The numerical results confirm the analytical predictions. The lateral shift under the lossy case is also discussed.

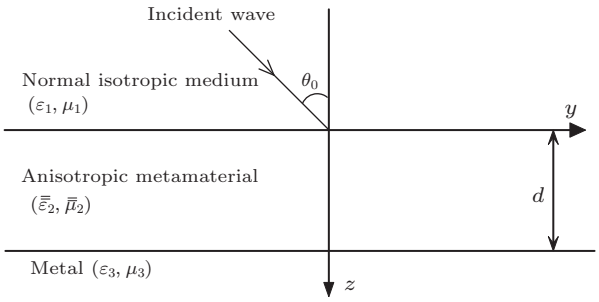

Fig. 1. Geometry of the beam incident upon an anisotropic metamaterial slab backed by a metal.

The geometry of the problem is shown in Fig. 1. Consider a two-dimensional ($\partial/\partial x = 0$) light beam of TE polarization (according to the principle of duality

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**Email: cm94@163.com

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the TE and TM cases are dual of each other, so we can easily obtain expressions for the TM case by interchanging ε and μ) and of angular frequency ω (corresponding to wavelength λ) incident from an ordinary isotropic medium upon an AMM slab backed by a metal at an oblique angle θ_0 with respect to the normal of the interface. The relative permittivity and permeability of the isotropic normal medium are ε_1 and μ_1. The slab's thickness is d and it is a biaxial medium of constitutive tensors $\bar{\bar{\varepsilon}}_2 = \varepsilon_{2x}\hat{x}\hat{x} + \varepsilon_{2y}\hat{y}\hat{y} + \varepsilon_{2z}\hat{z}\hat{z}$ and $\bar{\bar{\mu}}_2 = \mu_{2x}\hat{x}\hat{x} + \mu_{2y}\hat{y}\hat{y} + \mu_{2z}\hat{z}\hat{z}$. We assume that the electric field of the plane wave component of the incident light beam is $E_{\rm in} = E_0 \exp\left[i(k_y y + k_{1z} z)\right]$, where $k_y = k_1 \sin\theta_0$, $k_{1z} = k_1 \cos\theta_0$, $k_1^2 = \omega^2/c^2 \varepsilon_1 \mu_1 = (2\pi/\lambda)^2 \varepsilon_1 \mu_1$ is the wave number in the normal medium, and c denotes the speed of light in vacuum. We can derive the dispersion relations in normal isotropic media and the AMM, respectively,

$$k_{1z}^2 + k_y^2 = k_1^2, \tag{1}$$

$$k_{2z}^2 = \frac{\mu_{2y}}{\mu_{2z}}\left(\frac{\omega^2}{c^2}\varepsilon_{2x}\mu_{2z} - k_y^2\right). \tag{2}$$

It is noted that the choice of sign of the wave vector k_{2z} must ensure that the Poynting vector inside the AMM points away from the interface between the incident medium and the AMM; hence k_{2z} and μ_{2y} must have the same sign.[30]

For simplicity, we consider the metal layer as a perfect conductor (in Refs. [36,37] the similar assumption has been used). Based on Maxwell's equations and boundary conditions, the reflection coefficients from a TE incidence are given by[30,31]

$$R_{\rm slab} = \left(R_{\rm int} - e^{2i\phi}\right)/\left(1 - R_{\rm int} e^{2i\phi}\right) = \left|R_{\rm slab}\right| e^{i\Phi_r}, \tag{3}$$

where $R_{\rm int} = \left(\mu_{2y}k_{1z} - \mu_1 k_{2z}\right)/\left(\mu_{2y}k_{1z} + \mu_1 k_{2z}\right)$, $\phi = k_{2z}d$, $\left|R_{\rm slab}\right| = 1$, Φ_r denotes the phase of the reflection coefficients.

For a wide incident beam (i.e. with a very narrow angular spectrum $\Delta k \ll k$), the lateral shifts of the reflected beams can be expressed as

$$\Delta_r = -d\Phi_r/dk_y, \tag{4}$$

which was proposed by using the stationary phase method.[2] When the transverse wave number k_{2z} becomes imaginary, i.e., the total internal reflection occurs. According to Eq. (4), we have the lateral shift Δ_r expressed as

$$\Delta_r = \frac{2\mu_1 k_y (\omega/c)^2/k_{1z}\alpha_{2z}}{(\mu_1\alpha_{2z})^2 + (\mu_{2y}k_{1z})^2}\frac{\mu_{2y}}{\mu_{2z}}(\varepsilon_1\mu_1 - \varepsilon_{2x}\mu_{2z})\mu_{2y}, \tag{4a}$$

where $k_{2z} = i\alpha_{2z}$. When the transverse wave number k_{2z} is real, we can obtain

$$\tan\Phi_r = \frac{\sin(2k_{2z}d)k_{1z}k_{2z}\mu_1\mu_{2y}}{(k_{2z}^2\mu_1^2\cos^2(k_{2z}d) - k_{1z}^2\mu_{2y}^2\sin^2(k_{2z}d))}.$$

According to Eq. (4a), the analytical result of the lateral shift Δ_r of the reflected wave can be presented as

$$\Delta_r = \frac{2\mu_1 k_y d}{\mu_{2z}k_{1z}}\cdot\frac{(k_{2z}^2\mu_{2z}\mu_{2y} - k_{1z}^2\mu_{2y}^2)\frac{\sin(2k_{2z}d)}{2k_{2z}d} + k_{1z}^2\mu_{2y}^2}{k_{2z}^2\mu_1^2\cos^2(k_{2z}d) + k_{1z}^2\mu_{2y}^2\sin^2(k_{2z}d)}. \tag{4b}$$

Since the components of $\bar{\bar{\varepsilon}}_2$ and $\bar{\bar{\mu}}_2$ can be either positive or negative, the dispersion relation of the AMM, given in Eq. (2), can be either elliptic or hyperbolic according to different combinations of the parameters.[38] Four cases will be considered to demonstrate the relationship between the lateral shift and the anisotropic permittivity and permeability tensors. In this Letter, we consider only the property of the lateral shift for $\mu_{2z} < 0$, the result for $\mu_{2z} > 0$ is opposite to that for $\mu_{2z} < 0$.

Case 1: $\mu_{2y}/\mu_{2z} > 0$ and $\varepsilon_{2x}\mu_{2y} > 0$. When $\theta > \theta_c = \sin^{-1}\sqrt{\varepsilon_{2x}\mu_{2z}/\varepsilon_1\mu_1}$, the transverse wave number k_{2z} becomes imaginary, i.e., total internal reflection occurs. Then we can obtain one expression for the lateral shift which is the same as (4a). Since μ_1, k_y, k_{1z} and α_{2z} are positive, hence the sign of the lateral shift is determined by the parameter μ_{2y}. When $0 < \theta < \theta_c = \sin^{-1}\sqrt{\varepsilon_{2x}\mu_{2z}/\varepsilon_1\mu_1}$, the transverse wave number k_{2z} is real, and we find one expression for the lateral shift which is the same as (4b). Note that μ_1, k_y, k_{1z} are positive, hence the sign of the lateral shift is determined by the following factor:

$$f = \frac{1}{\mu_{2z}}\left[\left(k_{2z}^2\mu_{2z}\mu_{2y} - k_{1z}^2\mu_{2y}^2\right)\frac{\sin(2k_{2z}d)}{2k_{2z}d} + k_{1z}^2\mu_{2y}^2\right]. \tag{5}$$

It is seen that when inequality

$$\left(k_{2z}^2\mu_{2z}\mu_{2y} - k_{1z}^2\mu_{2y}^2\right)\frac{\sin(2k_{2z}d)}{2k_{2z}d} + k_{1z}^2\mu_{2y}^2 < 0, \tag{6}$$

holds, the sign of the lateral shift is opposite to that of μ_{2z}. Here the periodical functions over $k_{2z}d$ in Eq. (4b) can be viewed as the result of the interaction between the effects of the two interfaces in the present situation. It is interesting to note that the oscillation of the lateral shift with respect to d is closely related to the periodical occurrence of resonance at $k_{2z}d = m\pi (m = 1, 2, 3, \ldots)$.

Case 2: $\mu_{2y}/\mu_{2z} < 0$ and $\varepsilon_{2x}\mu_{2y} < 0$. When $0 < \theta < \theta_c = \sin^{-1}\sqrt{\varepsilon_{2x}\mu_{2z}/\varepsilon_1\mu_1}$, the transverse wave number k_{2z} is imaginary, we can now derive an expression for the lateral shift which is identical to (4a). When $\theta > \theta_c = \sin^{-1}\sqrt{\varepsilon_{2x}\mu_{2z}/\varepsilon_1\mu_1}$, the wave vector k_{2z} is real, we can obtain an expression for the lateral shift which is the same as (4b).

Case 3: $\mu_{2y}/\mu_{2z} < 0$ and $\varepsilon_{2x}\mu_{2y} > 0$. In this case, the wave vector k_{2z} is always real. Any waves impinging the anisotropic region will be changed into propagating waves no matter whether the incident waves are propagating or evanescent. Hence there will be no

reflected beam shift for all incident angles.

Case 4: $\mu_{2y}/\mu_{2z} > 0$ and $\varepsilon_{2x}\mu_{2y} < 0$. Due to the imaginary k_{2z} in the anisotropic region all incident beams are totally reflected, i.e., the AMM here behaves like the electric or magnetic plasma. In this case, the lateral shift of the reflected wave always exists for any incident angles.

For simplicity, we assume $\varepsilon_1 = \mu_1 = 1$. The dependence of the lateral shift of the beam on the incident angle is shown in Fig. 2. Figure 2(a) shows the dependence of the reflected beam shift on the incidence angle in case 1, where k_{2z} is real, $\varepsilon_{2x} = -1.0$, $\mu_{2y} = -0.5, \mu_{2z} = -1.8$. The sign of the lateral shift is identical to that of μ_{2z} when the incident angle is not large. For a large angle of incidence the sign of the shift is opposite to that of μ_{2y} and the lateral shift becomes larger with an increasing angle of incidence. It is clear that the similar dependence can be found in cases 2 and 3 when k_{2z} is real. When the incidence angle tends to $\pi/2$, the lateral shift approaches infinity.

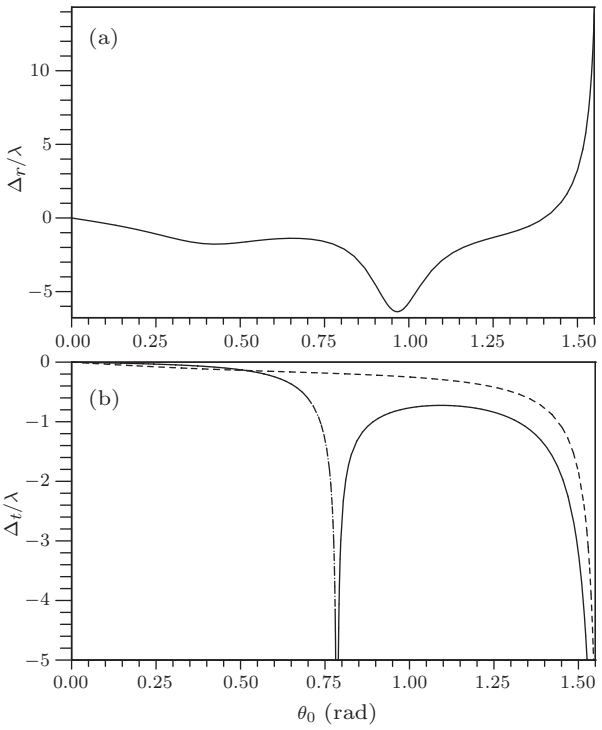

Fig. 2. Relation between lateral shift and incident angle. (a) The transverse wave number k_{2z} is real: $\varepsilon_{2x} = -1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = -1.8$; (b) the transverse wave number k_{2z} is imaginary: solid line for $\varepsilon_{2x} = -1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = -1.0$, dot-dashed line for $\varepsilon_{2x} = 1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = 1.0$, dashed line for $\varepsilon_{2x} = 1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = -1.0$. Here the thickness of the slab is $d = 4\lambda$.

A typical dependence of the reflected beam shift on the incident angle is shown in Fig. 2(b), where k_{2z} is imaginary. The solid line in Fig. 2(b) corresponds to case 1, in which $\varepsilon_{2x} = -1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = -1.0$. The lateral shift appears when the incident angle is greater than the critical angle θ_c. Since the stationary phase theory fails near the critical angle, the shift becomes divergent near θ_c.[2] For case 2 in which $\varepsilon_{2x} = 1.0, \mu_{2y} = -0.5$, $\mu_{2z} = 1.0$, the beam shift is restricted in the range $0 < \theta_0 < \theta_c$, as we can see from the dot-dashed line in Fig. 2(b). For case 4, as shown by the dashed line in Fig. 2(b), the lateral shift exists for all incident angles and increases with respect to incident angle θ_0, where $\varepsilon_{2x} = 1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = -1.0$. Here we can clearly see that the sign of shift is consistent with that of μ_{2z} for each case in Fig. 2(b).

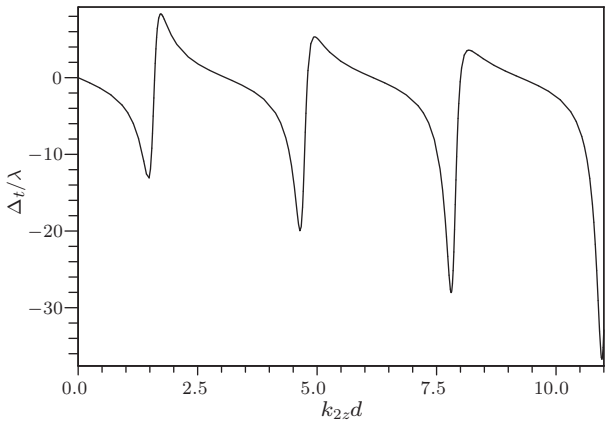

Fig. 3. Dependence of transmitted beam shift on the thickness d, where k_{2z} is real, $\varepsilon_{2x} = -1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = -1.8$, the incident angle $\theta_0 = 1.45$ rad.

A typical dependence of the reflected beam shift on the slab's thickness d is shown in Fig. 3, where k_{2z} is real, $\varepsilon_{2x} = -1.0$, $\mu_{2y} = -0.5$, $\mu_{2z} = -1.8$, the incident angle $\theta_0 = 1.45$ rad. We can find that the oscillation of the lateral shift about d is closely related to the periodical occurrence of resonances. According to Eq. (4b), when k_{2z} is real, similar phenomenon can also occur in cases 2 and 3. When k_{2z} is imaginary, we can find from Eq. (4a) that the lateral shift is independent of the thickness d.

Most realistic materials are dissipative, so it is necessary to consider the lateral shift from an absorbing AMM slab. For a lossless metamaterial slab, the lateral shift of the reflected beam has no definition at resonance ($2k_{2z}d = m\pi$, where m is an integer) because the reflection coefficient vanishes near the resonance. When k_{2z} is real, we can obtain

$$\Phi_r = \arctan\left[\frac{\sin(2k_{2z}d)k_{1z}k_{2z}\mu_1\mu_{2y}}{k_{2z}^2\mu_1^2\cos^2(k_{2z}d) - k_{1z}^2\mu_{2y}^2\sin^2(k_{2z}d)}\right]. \tag{7}$$

From Eq. (6), we can see that there is a sudden π phase change across the resonances (i.e., an infinite slope accompanied by null reflection) that occurs at $2k_{2z}d = m\pi$. In view of continuity it is reasonable to expect a large finite slope of the phase change with a nonzero reflection, if the AMM slab is weakly absorb-

ing. According to Eq. (4), we can see that the phase change leads to the large lateral shift near resonance with a nonzero reflection. It is clear that near the resonance the absorption of AMM slab decreases while the lateral shift of the reflected beam becomes larger (owing to the steeper phase change). That is to say, if the absorption of the slab becomes smaller and smaller, but is not zero, the lateral shift near resonance will go to a delta function.

These properties are similar to those discussed in Refs. [39,40], where the slab was a weakly absorbing dielectric medium or isotropic left-handed metamaterial. When k_{2z} is imaginary, the lateral shift of the reflected beam behaves similarly to the lossless case.

In summary, we have theoretically investigated the lateral shift of a light beam incident at the surface of an anisotropic metamaterial slab backed by a metal. By using the stationary phase theory, expressions for the lateral shift are obtained and the conditions for the lateral shift to be positive or negative are identified. Furthermore, the properties of the lateral shift are analysed for four different anisotropic metamaterial. Dependences of the lateral shift on the thickness and physical parameters of the anisotropic metamaterial slab and the incident angle are discussed. Finally we study the lossy effect of the anisotropic metamaterial on the lateral shift.

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