

Potential Energy of the Di-nuclear System *

JIA Fei(贾飞)^{1,2}, XU Hu-Shan(徐瑚珊)¹, HUANG Tian-Heng(黄天衡)^{1,2}, LI Wen-Fei(李文飞)¹,
XU Hua-Gen(徐华根)^{1,2}, CHEN Ruo-Fu(陈若富)^{1,2}, LI Jun-Qing(李君清)^{1**}

¹*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000*

²*Graduate School of the Chinese Academy of Sciences, Beijing 100039*

(Received 1 February 2005)

We perform the calculation of the nucleus–nucleus interaction potential of the dinuclear system with deformed nuclei. Based on the calculated results by properly treating the double-folding method, some results from the analytical expressions for calculating both the nuclear and the Coulomb interactions are investigated. It is concluded that the analytical formula to calculate the Coulomb interaction by Wong [Phys. Rev. Lett. 31 (1973) 766] can reproduce good double folding results. However the results by the parametrized Morse formula, which are used to calculate the nuclear interaction, greatly deviate from the double folding results, unless the distance of nuclei is adjusted to be shifted to a smaller relative distance determined by the same distance between the nuclear surfaces as one of them pertains to the deformed nuclei. Using the double folding method to calculate the nuclear interaction and employing Wong’s analytical formula to calculate the Coulomb interaction can keep good precision and can cost much less computation time.

PACS: 27.90.+b, 21.30.Fe, 21.30.-x

The formation of compound nuclei by fusion of heavy nuclei is currently used to synthesize the super heavy nuclei (SHN).^[1,2] Reactions between heavy nuclei may be described with the dinuclear system (DNS) concept.^[3,4] The dinuclear system is a configuration of two touching nuclei which keep their individuality, and fusion process is considered as the evolution of the DNS in which nucleons or clusters are transferred from the light nucleus to the heavy one. The dynamics has been treated as a diffusion in mass asymmetry at the touching point to the compound nucleus, and in the variable R of the relative distance between the centres of the interacting nuclei, which may lead to the quasi-fission. The analytical solution of the Fokker-Planck equation^[4,5] or the numerical solution of the Master equation^[6,7] are used to describe the diffusion process. In both the cases, the potential energy of the DNS is of vital importance.

It is estimated^[8] that the overlap volume of the colliding nuclei in the DNS is a few percent of the total DNS volume, thus the potential energy $V(R, \eta, J)$ (where $\eta = (A_1 - A_2)/(A_1 + A_2)$ is the mass asymmetry parameter with A_1 and A_2 being the mass numbers of the DNS.) can be written as the sum of the binding energies of two nuclei (B_1 and B_2) and the energy of their interaction $U(R, A_1, A_2)$. The nucleus–nucleus potential $U(R, A_1, A_2)$ is the sum of nuclear $U_N(R, A_1, A_2)$, Coulomb $U_{\text{coul}}(R, A_1, A_2)$ and centrifugal $U_{\text{rot}}(R, A_1, A_2)$ potentials. Here we restrict ourselves to head on collisions, the rotational energy of the compound nucleus can be ignored, and

it can be added straightforward otherwise. Therefore the calculation of the potential energy $V(R, A_1, A_2)$ is reduced to the calculation of the nucleus–nucleus interaction in the DNS. Various methods have been introduced for calculating the interaction potentials between two nuclei.^[8–10] The main point is the calculation of the nucleus–nucleus interaction $U_N(R, A_1, A_2)$ due to the uncertainties of the nuclear interaction. Some double folding procedures with different choices of the nucleon-nucleon interaction,^[8–10] and some parameterized analytical expressions^[5] are proposed. If nuclei are deformed, the deformation and the corresponding orientation will bring some additional complications. Some analytical expressions may closely simulate the result of the double folding one, which are convenient to use and can save much calculation time. In the present work we investigate and compare different ways to calculate the potential energy of DNS and to check the precision of the calculated potential energy which has been used in Ref. [6].

The potential energy of the DNS can be written as

$$U(A_1, A_2, R) = B_1 + B_2 - B_{12} + U_{\text{coul}}(A_1, A_2, R) + U_N(A_1, A_2, R), \quad (1)$$

where B_1 , B_2 , and B_{12} are the binding energies of the fragments and compound nucleus, respectively. The shell and pairing corrections are included in the binding energies. If the ground state deformations of the two touching nuclei are taken into account, in principle, the deformed nuclei can have different relative ori-

* Supported by the National Natural Science Foundation of China under Grant Nos 10175082, 10235020 and 10235023, the Major Basic Research Development Programme under Grant No G2000-0774-07, the Knowledge Innovation Project of CAS under Grant Nos KJCX2-SW-N02 and KJCX2-SW-N04, One Hundred Person Project of Chinese Academy of Sciences, the National Key Basic Research and Development Programme of China under Grant Nos 2001CCB01200 and 2002CCB00200, and the DFG of Germany.

** To whom correspondence should be addressed. Email: jqli@impccas.ac.cn

entations. Some averaging over the orientations of the nuclei has to be carried out in the initial DNS; however, the orientation which gives rise to the minimum interaction energy is in favour of the nucleon transfer, so the pole-to-pole orientation is chosen hereafter since it gives rise to the minimum energy.^[10,11]

The Coulomb interaction $U_C(A_1, A_2, R)$ of the deformed nuclei in the DNS usually should be calculated numerically as by

$$U_C(R) = \rho_1^0 \rho_2^0 \int \frac{d\mathbf{r}_1 d\mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{R}|}, \quad (2)$$

where \mathbf{R} is the vector between the two centres of the nuclei. The charge densities are assumed to be constants. The symmetry axes of the two deformed nuclei and the z -axis are assumed to be in the same plane. On the other hand, an analytical formula from Ref. [12] reads

$$\begin{aligned} U_{\text{coul}}(R, \theta) = & \frac{Z_1 Z_2 e^2}{R} + \left(\frac{9}{20\pi}\right)^{1/2} \left(\frac{Z_1 Z_2 e^2}{R^3}\right) \\ & \cdot \sum_{i=1}^2 \Re_i^2 \beta_2^{(i)} P_2(\cos \theta_i) \\ & + \left(\frac{3}{7\pi}\right)^{1/2} \left(\frac{Z_1 Z_2 e^2}{R^3}\right) \\ & \cdot \sum_{i=1}^2 \Re_i^2 [\beta_2^{(i)} P_2(\cos \theta_i)]^2, \end{aligned} \quad (3)$$

where θ_i is the angle measured between the radius vector \mathbf{R} and the symmetry axis of the i th nucleus. \Re_i is the radius of the deformed nucleus i . In the case with the pole-to-pole orientation, we have $\theta_i = 0$.

For the nuclear potential energy, adopting a Skyrme-type interaction without considering the momentum and spin dependence it is formulated as follows:

$$\begin{aligned} U_N(R) = & C_0 \left\{ \frac{F_{in} - F_{ex}}{\rho_{00}} \left(\int \rho_1^2(\mathbf{r}) \rho_2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \right. \right. \\ & + \int \rho_1(\mathbf{r}) \rho_2^2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \\ & \left. \left. + F_{ex} \int \rho_1(\mathbf{r}) \rho_2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \right\}, \end{aligned} \quad (4)$$

with

$$F_{in,ex} = f_{in,ex} + f'_{in,ex} \frac{N_1 - Z_1}{A_1} \frac{N_2 - Z_2}{A_2}, \quad (5)$$

where a zero-range treatment of the effective interaction $\delta(\mathbf{r}_1 - \mathbf{r}_2)$ is assumed. The nuclear potential is obtained in the sudden approximation.^[10] $N_{1,2}$ and $Z_{1,2}$ are the neutron and proton numbers of the two nuclei, respectively. The parameters $C_0 = 300 \text{ MeV} \cdot \text{fm}^3$, $f_{in} = 0.09$, $f_{ex} = -2.59$, $f'_{in} = 0.42$, $f'_{ex} = 0.54$, and $\rho_{00} = 0.17 \text{ fm}^{-3}$ are used. The functions ρ_1 and ρ_2 are the two-parameter Woods–Saxon density distributions.

The nuclear interaction energy parameterized by the Morse potential reads^[5]

$$U_N(A_1, A_2) = D \left(\exp \left[-2\alpha \frac{R - R_0}{R_0} \right] - 2 \exp \left[-\alpha \frac{R - R_0}{R_0} \right] \right), \quad (6)$$

where $D = 2\pi a_1 a_2 R_{12} (10.96 - 0.8 R_{12})$ (in MeV), $R_0 = R_1 + R_2$, and $\alpha = 11.47 + 2.069 R_{12} - 17.32 a_1 a_2$ (dimensionless) are the depth, minimum position, and inverse width of the potential, respectively; $R_{12} = R_1 R_2 / R_0$ (R_1, R_2 are the radii of the nuclei). Here $a_1, a_2 \approx 0.54 - 0.59$ and R is the distance between the centres of nuclei.

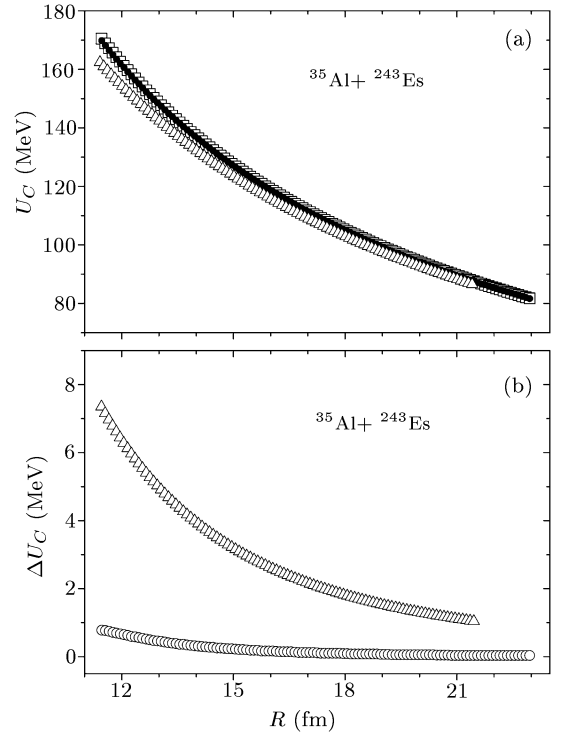


Fig. 1. (a) The Coulomb interaction U_C of the system $^{35}\text{Al} + ^{243}\text{Es}$ calculated by the double folding integration (open squares) and by the analytical expression (solid circles) as a function of the distance R between the centres of nuclei, respectively; as well as the Coulomb interaction of the system $^{35}\text{Al} + ^{243}\text{Es}$ without considering the deformation (triangles). (b) Difference between the two results (circles) and the difference between the results by the analytical expression with and without the deformation (triangles).

The Coulomb interaction U_C of the system $^{35}\text{Al} + ^{243}\text{Es}$ calculated by the double folding integration formula Eq. (2) (C-INT) and by the analytical expression (3) (C-ANA) as a function of the distance R between centres of the nuclei are shown in Fig. 1(a) with open squares and solid dots, respectively. The ground state quadrupole deformations of ^{35}Al and ^{243}Es are $\beta_2 = 0.216$, and 0.224 , respectively. Hereafter all the deformation parameters are taken from Ref. [13]. Two

results are nearly overlapping. The difference between the two results are shown in Fig. 1(b) by circles. The curve by open triangles in Fig. 1(a) is the Coulomb interaction of the system $^{35}\text{Al} + ^{243}\text{Es}$ without considering the deformation, for which C-INT and C-ANA give the same results, and which is lower than the result with deformation. The difference between the results by the analytical expression C-ANA with and without the deformation is indicated in Fig. 1(b) by triangles.

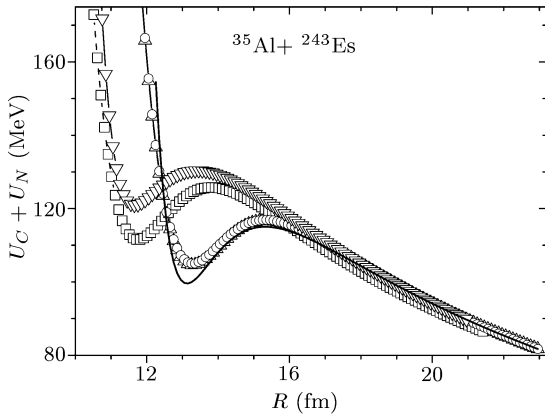


Fig. 2. The nuclear plus Coulomb interaction potential of the system $^{35}\text{Al} + ^{243}\text{Es}$ calculated by different ways as a function of R . The curve with open circles represents the calculation by the double folding integrations of Eq. (4) (N-INT) and C-INT. The curve with up-triangles denotes the results by N-INT and C-ANA. The dashed line with open squares shows the calculation by N-INT+C-ANA but without considering the deformation. The curve with down-triangles denotes the calculation by N-ANA+C-INT. The nuclear plus Coulomb interaction potential of the system (N-ANA(M)+C-INT) is shown by the solid line.

The nuclear plus Coulomb interaction potential of the system $^{35}\text{Al} + ^{243}\text{Es}$ calculated by different ways as a function of R is shown in Fig. 2, in which the curve with open circles represents the calculation by the double folding integrations of Eq. (4) (N-INT) and C-INT, which is thought of as the most precise result of nuclear plus Coulomb interaction of the system presently considered. However, the six-fold integrations for the Coulomb interaction take great computation time. The curve with up-triangles denotes the results by N-INT and C-ANA, which largely overlaps with N-INT+C-INT. Since the Coulomb interaction by C-INT largely overlaps with that by the analytical expression C-ANA, it reduces the integration to three-fold in a good precision. In Fig. 2, the dashed line with open squares shows the calculation by N-INT+C-ANA but without considering the deformation. The nuclear interaction energy parameterized by the Morse potential of Eq. (6) (N-ANA) was widely used, e.g. by Adamian *et al.*^[5] In Fig. 2, the curve with down-triangles denotes the calculation by

N-ANA+C-INT, which is highly above all the curves. The parameterized Morse potential Eq. (6) (N-ANA) does not take account the deformation. As compared it with the squares, the deformation in C-INT increases the Coulomb potential energy, and the parameterized Morse potential formula underestimates the minus nucleus-nucleus interaction. However, the strong short range interaction character keeps the position of the potential pocket unmoved. In Ref. [6] the parametrized Morse formula was adopted in such a way that two nuclei were assumed to be spherical but shifted to a smaller relative distance determined by the same distance between the nuclear surfaces as the one of them pertains to the deformed nuclei. In this manner the deformation of the nuclei is simulated in the nuclear part of the potential, and here we call the calculation N-ANA(M). If the Coulomb interaction was calculated by C-INT, the nuclear plus Coulomb interaction potential of the system (N-ANA(M)+C-INT) is shown by the solid line in Fig. 2. At the bottom of the pocket it is lower than the result of N-INT+C-INT for about 4 MeV due to larger deformations of Al and Es. Obviously, in this way the nucleus-nucleus interaction has been overestimated. Compared to the other case the deviation is not too large.

The potential energy governing the nucleon transfer in the DNS is calculated by Eq. (1) as a function of the parameter η . Here R is not taken as an independent variable in the current calculation, but as $R = R_1 + R_2 + R_d$, where R_d is chosen as the value which gives the minimum value of $U_C(A_1, A_2) + U_N(A_1, A_2)$. For the system $^{70}\text{Zn} + ^{208}\text{Pb}$, the calculation of the potential energy starts from the entrance configuration (Zn, Pb) = (A_p, A_t) , i.e. $\eta_i = (A_p - A_t)/(A_p + A_t)$. The one-nucleon transfer from η_i to both the sides, no matter whether it is a neutron or a proton, depends on which direction the potential energy is lower. Consequently, the driving potential of Eq. (1) is an explicit function of neutron and proton numbers of fragments. In Figs. 3(a) and 3(b) full circles denote the results by N-INT+C-INT, and the ground-state quadrupole deformations of nuclei are considered. The solid line in Fig. 3(a) is by N-INT+C-ANA. Since the analytical calculation of the Coulomb interaction well reproduce the double folding integration, two curves basically overlap with each other except in the region near $\eta = 0$. Due to our above-mentioned selection rule of the isotopic composition, the small difference from the two calculations makes the two cases take different N/Z ratios while passing over $\eta = 0$. Open circles in Fig. 3(a) are the results by N-INT+C-ANA without considering the deformation. From Fig. 1, it is obvious that the deformation increases the Coulomb interaction energy a little, but the deformation decreases the nuclear interaction potential energy. The larger deviations between the full circles and open ones appear in the regions of $|\eta| = 0.03 \sim 0.4$ and $|\eta| = 0.7 \sim 0.9$,

where nuclei in the DNS have relatively larger deformations. Therefore the deformation can greatly reduce the interaction energy of the DNS, which is more than the role played by the shell effect. In Fig. 3(b) open circles are by N-ANA+C-INT without considering the deformation. It is again above the full circles by N-INT+C-INT. The asterisks are by N-ANA+C-INT. The difference between the open circles and asterisks are from the Coulomb integration with and without considering the deformation, and moreover from the different isotopic composition between the two cases. The solid line is by N-ANA(M)+C-INT, which is closest to the precise N-INT+C-INT among the others. There are some deviations at $|\eta| = 0.1 \sim 0.3$ and $|\eta| = 0.7 \sim 0.9$ due to the relatively larger deformations in the regions, and the different parameters used in the two methods. The deviation magnitude and the range are smaller than those between N-ANA+C-INT with (asterisks) and without (open circles) considering the deformation, since here the deformation has been simulated although it has not been precisely treated. However, the simulation of the deformation overestimated the nuclear interaction energy.

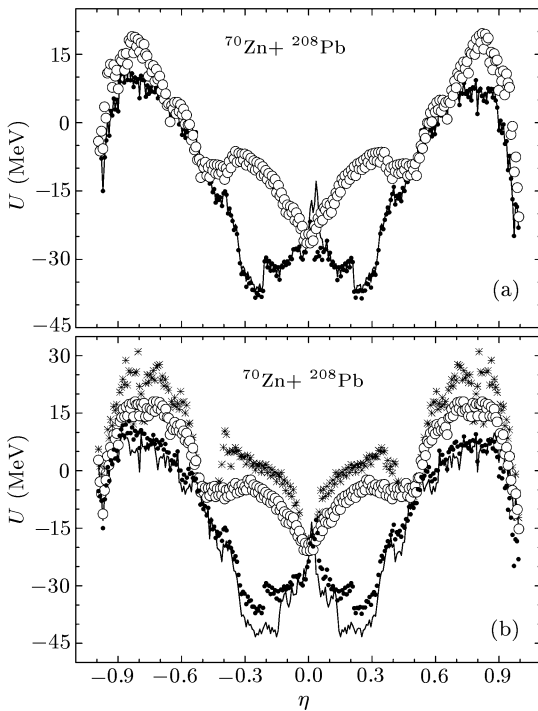


Fig. 3. The potential energy governing the nucleon transfer in the DNS shown as a function of the mass parameter η for the system $^{70}\text{Zn} + ^{208}\text{Pb}$. Closed circles in (a) and (b) denote the results by N-INT+C-INT. In (a) the solid line and open circles are by N-INT+C-ANA with and without deformation respectively. In (b) asterisks and open circles are the results by N-ANA+C-INT with and without deformation respectively, and the solid line is by N-ANA(M)+C-INT.

The nucleus–nucleus interaction potential is very important for studying the formation and the evolution of the DNS. For the DNS with deformed nuclei there are some additional complications for calculating the nuclear and Coulomb interaction. A better way is the properly treated double folding method, which brings with it, however, a very long computation time. Therefore, some used analytical expressions have been checked and compared with the double folding treatment. It is concluded that the analytical formula to calculate the Coulomb interaction by Wong^[12] can well reproduce the double folding results. However, the results by the parametrized Morse formula, which is used to calculate the nuclear interaction, greatly deviate from the double folding results, unless the distance of nuclei is adjusted to be shifted to a smaller relative distance determined by the same distance between the nuclear surfaces as one of them pertains to the deformed nuclei. This is an exact way that Ref. [6] has used, and here it is indicated that the method is basically suitable. Using the double folding method to calculate nuclear interaction and Wong’s analytical formula to calculate Coulomb interaction can keep a good precision and can take much less computation time. However, one must pay attention to the isotopic composition of the DNS, which is not difficult to handle. Furthermore, the fusion probability calculated by numerically solving the Master equation by using the driving potential indicates that the difference at the region nearby $\eta = 0$ does not change the fusion probability appreciably.

References

- [1] Hofmann S 1998 *Rep. Prog. Phys.* **61** 636
- [2] Oganessian Y T, Yeremin A V, Popeko A G et al 1999 *Nature* **400** 242
- [3] Volkov V V 1986 *Izv Akad. Nauk SSSR Ser. Fiz.* **50** 1879
- [4] Antonenko N V, Cherepanov E A, Nasirov A V et al 1993 *Phys. Lett. B* **319** 425
- [5] Adamian G G, Antonenko N V and Scheid W 1997 *Nucl. Phys. A* **618** 176
- [6] Li W F, Wang N, Li J F et al 2003 *Europhys. Lett.* **64** 750
- [7] Diaz-Torres A, Adamian G G, Antonenko N V et al 2001 *Phys. Rev. C* **64** 024604
- [8] Adamian G G, Antonenko N V, Jolos R V et al 1996 *Int. J. Mod. Phys. E* **5** 191
- [9] Denisov V Y and Nörenberg W 2002 *Eur. Phys. J. A* **15** 375
- [10] Li Q F, Zuo W, Li W F et al 2005 *Eur. Phys. J. A* **24** 223
- [11] Wang N, Zhao E G and Li J F 2003 *Chin. Phys. Lett.* **20** 635
- [12] Wong C Y 1973 *Phys. Rev. Lett.* **31** 766
- [13] Möller P, Nix J R, Myers W D et al 1995 *At. Data Nucl. Data Tables* **59** 185