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Faraday Effect on Negative Refraction in Uniaxial Anisotropic Plasma Metamaterials *

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Negative refraction is the name for an electromagnetic phenomenon in which light rays are refracted at an interface in the reverse sense to that normally expected. A uniaxial anisotropic plasma metamaterial that exhibits negative refraction is demonstrated and the necessary conditions are derived for negative refraction. The Faraday effect on the negative refraction in the proposed plasma metamaterials is discussed. Parameter dependences such as plasma filling factor, dielectric constant of background materials, and external magnetic field are calculated and discussed.

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Great efforts have been made to study and understand photonic crystals (PCs) and metamaterials, in which two different types of materials are being evolved in the area of photonics.[1,2] PCs are periodic optical nanostructures that affect the motion of photons in the same way that ionic lattices affect the electrons in solids.[3,4] Photons (behaving as waves or light) propagate through the nanostructures depending or not depending on their wavelengths. The wavelengths of light that are allowed to travel are known as the modes, and the groups of allowed modes form bands. Disallowed bands of wavelengths are called photonic band gaps. Another more development in the field of photonics is metamaterials,[5−9] which are artificial materials engineered to have properties that may not be found in nature. Metamaterials are assemblies of multiple individual elements fashioned from conventional microscopic materials such as metals or plastics, while the materials are usually arranged in periodic patterns. The technical applications of PCs and metamaterials are now expanding widely in many areas such as antennae, sensors, and cloaking because of their novel properties.

In the past few years, plasma PCs[10−24] and plasma metamaterials[25−28] have drawn significant scientific interest and are intensively analyzed because of the outstanding feature of plasmas. Plasmas, which were recently reported both in plasma PCs and plasma metamaterials, have advantages over ordinary materials because a dynamic change of permittivity and tunable amplitude on the complex plane can be manipulated by an external power supply for plasma generation and adjustable gas pressure or temperature. These two studies have also been applied in the area of interaction of electromagnetic waves and plasmas for fundamental studies as well as for potential applications in many areas.[29−34]

Negative refraction is an electromagnetic (EM) phenomenon in which light rays are refracted at an interface in the reverse sense to that normally expected. It occurs when a beam of light is refracted at an interface, somewhat unexpectedly at first glance, not into the usual quarter-space seen in diagrams in textbooks on electromagnetics and optics, but into the other quarter-space left blank in those diagrams. As first proposed, the permittivity ε and permeability μ were required to be simultaneously less than zero.[5] The negative value of ε is a natural property of metals and plasmas and therefore negative-ε metamaterials can be created by simple means such as the addition of metallic rods or microplasmas, in which the negative value of μ is obtained by using a resonance.[6,7] However, this double resonance scheme faces limitations because the design and fabrication can be complicated. Fortunately, one study suggested that a uniaxial anisotropic metamaterial, in which μ is scalar and positive and only the two principal values of ε have different signs, can be observed to have interesting properties for materials with simultaneously negative ε and μ.[8] Motivated by this, in our previous work,[28] we have demonstrated a uniaxial anisotropic plasma metamaterial to explore the negative refraction in the terahertz region. However, in Ref. [28], we only focus on unmagnetized plasma. It is well known that magnetized plasma has different properties from unmagnetized plasma, while the properties of plasma have significant effects on negative refraction. Therefore, it is very necessary to study the properties of negative refraction in uniaxial anisotropic magnetized plasma metamaterials. It is of great theoretical significance to control the properties of plasma metamaterials in engineering applications by choosing the ex-

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ternal magnetic field. In this Letter, we discuses the Faraday effect on the negative refraction in unaxial anisotropic plasma metamaterials.

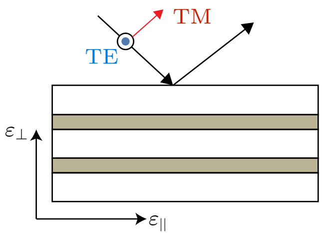

Fig. 1. The structure of the proposed uniaxial anisotropic plasma metamaterials and the relative orientation of the dielectric function $\varepsilon_\perp$ and $\varepsilon_\parallel$.

According to Refs. [9,28], a uniaxial anisotropic material (as shown in Fig. 1) with

$$\varepsilon_\perp < 0, \quad \varepsilon_\parallel > 0 \tag{1}$$

will exhibit negative refraction behavior for the transfer magnetic (TM) polarization for all incidence angles, while the transfer electric (TE) polarization does not experience anisotropy, both $\boldsymbol{k}$ and $\boldsymbol{S}$ refract normally. Here $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the components of the permittivity relative to the surface of the material.

For the Faraday effect, the propagating direction of EM waves is along the external magnetic field direction. A rotation of the plane of polarization of EM waves is expected due to the difference between the dielectric constants of the left- and right-handed circular polarizations. Then the different dielectric functions of the lossless plasma layer under the Faraday effect can be given by

$$\varepsilon_p = \varepsilon_\pm = 1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_{\rm ce})}, \tag{2}$$

where the subscripts + and − stand for right circular polarization (RCP) and left circular polarization (LCP), ω is the frequency of incident waves, ω_p is bulk plasma frequency, $\omega_{\rm ce} = eB/m$ is the cyclotron frequency of an electron and B is the external magnetic field. Both plasma and background materials are assumed to be non-magnetic materials and the permeability μ is set to 1.

Using the effective medium approximation,[35–37] we can obtain the expressions of the principal dielectric function as follows:

$$\varepsilon_\perp = \frac{\varepsilon_p \varepsilon_d}{\rho\varepsilon_d + (1-\rho)\varepsilon_p}, \tag{3}$$

$$\varepsilon_\parallel = \rho\varepsilon_p + (1-\rho)\varepsilon_d, \tag{4}$$

where ε_d is dielectric constant of the background materials, ε_p is the plasma dielectric function which is described in Eq. (2), and we also define $\rho = \frac{d_p}{d_p+d_b}$ ($\rho \in (0,1)$) as the relative filling factor of plasma in the unit cell, here d_p and d_b denote the thickness of plasma layer and background materials, respectively.

Figure 2 shows the calculated components of the dielectric function, $\varepsilon_\perp$ and $\varepsilon_\parallel$, both for RCP and LCP. Here for simplicity, we introduce the dimensionless variable ω_p. The parameters are selected as follows. The plasma filling factor $\rho = 0.5$, the external magnetic field $\omega_{\rm ce} = 1$ and the dielectric constant of the background material $\varepsilon_d = 5$. The spectral region where the plasma metamaterial will exhibit the negative refraction is indicated by the shaded regions: green for the LCP case and yellow for the RCP.

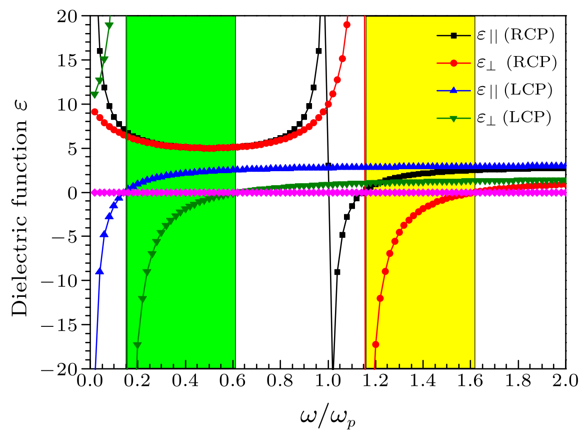

Fig. 2. The dielectric function, $\varepsilon_\perp$ and $\varepsilon_\parallel$, of the layered structure calculated using the effective medium approximation for both RCP and LCP cases. The shaded regions (green for LCP case and yellow for RCP) indicates the spectral region where the plasma metamaterial exhibits the negative refraction. We consider the parameters $\rho = 1/2$, $\omega_{\rm ce}$=1, and $\varepsilon_d = 5$.

From Eqs. (1)–(4), we note that there are four selective parameters, i.e. plasma frequency ω_p, plasma filling factor ρ, external magnetic field $\omega_{\rm ce}$, and dielectric constant of background materials ε_d, which determine the negative refraction. Here we focus on how to search a suitable plasma which leads to negative refraction. Before investigating the negative refraction, we firstly introduce the dimensionless variables, which are expressed as $\omega = \omega/\omega_p$ and $\omega_{\rm ce} = \omega_{\rm ce}/\omega_p$. By substituting Eqs. (3) and (4) in Eq. (1), we can obtain the necessary conditions for negative refraction in the proposed uniaxial anisotropic plasma metamaterials, which are expressed as

$$\frac{P\omega_{\rm ce} + \sqrt{\omega_{\rm ce}^2 + \frac{4}{[\frac{\rho}{1-\rho}\varepsilon_d + 1]}}}{2} < \omega < \frac{P\omega_{\rm ce} + \sqrt{\omega_{\rm ce}^2 + 4}}{2}, \quad 0 < \rho < \frac{1}{2}, \tag{5a}$$

$$\frac{P\omega_{\rm ce} + \sqrt{\omega_{\rm ce}^2 + \frac{4}{[\frac{1-\rho}{\rho}\varepsilon_d + 1]}}}{2} < \omega < \frac{P\omega_{\rm ce} + \sqrt{\omega_{\rm ce}^2 + 4}}{2}, \quad \frac{1}{2} \leqslant \rho < 1, \tag{5b}$$

where $P = -1$ for the LCP case and $P = 1$ for the RCP case.

From Eq. (5), we can derive the values of the critical frequency ω_0, which means where the negative refraction begins to occur, and also the bandwidth $\Delta\omega$, which means the region of negative refraction.The effects of plasma filling factor ρ, the external magnetic field $\omega_{\rm ce}$, and the dielectric constant ε_d on the critical frequency ω_0 and the bandwidth $\Delta\omega$, are shown in Figs. 3 and 4. It is clear that the effects of plasma filling factor ρ, the external magnetic field $\omega_{\rm ce}$, and dielectric constant ε_d have significant effects on the critical frequency ω_0 and the bandwidth $\Delta\omega$.

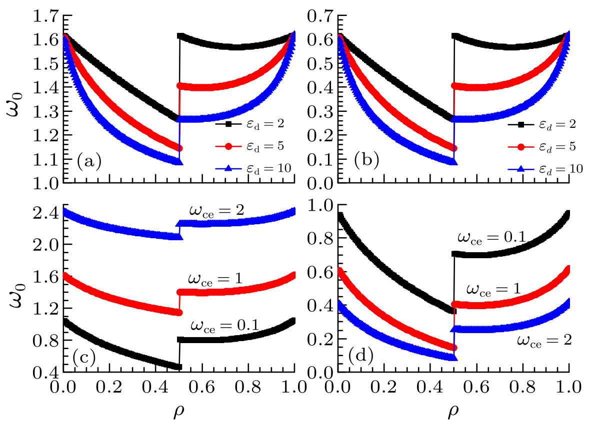

Fig. 3. The critical frequency ω_0 as a function of plasma filling factor ρ for three different dielectric constants ε_d [(a) and (b)] and three different external magnetic fields $\omega_{\rm ce}$ [(c) and (d)] in the LCP case [(a) and (c)] and in the RCP case [(b) and (d)].

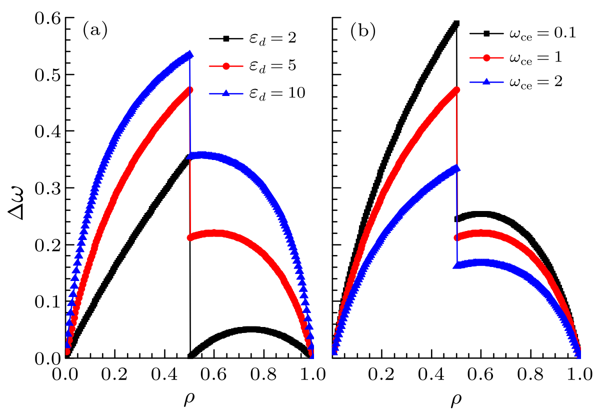

Fig. 4. The bandwidth $\Delta\omega$ as a function of plasma filling factor ρ for three different dielectric constant ε_d [(a)] and three different external magnetic field $\omega_{\rm ce}$ [(b)]. Here it is noticeable from Eq. (5) that the bandwidths in both the RCP and LCP cases are the same.

Figure 3 shows the effects of ε_d and $\omega_{\rm ce}$ on the behavior of critical frequency ω_0. It is noticeable that the critical frequency exhibits different behavior in different ρ regions in both the RCP and LCP cases. From Fig. 3, it is obvious that the quantity ω_0 first decreases then increases at the critical point $\rho = 1/2$. Moreover, the critical frequency decreases with the increase of dielectric constant ε_d for fixed plasma filling factor ρ in both the RCP and LCP cases, see Figs. 3(a) and 3(b). However, the effect of the external magnetic field on the behavior of critical frequency for the RCP and LCP cases are opposite: the quantity ω_0 increases with the external magnetic field $\omega_{\rm ce}$ in the LCP case, while ω_0 decreases in the RCP case, see Figs. 3(c) and 3(d). It is also clear that the value of the critical frequency ω_0 in the LCP case is more than that in the RCP case. The effects of ε_d and $\omega_{\rm ce}$ on the behavior of bandwidth $\Delta\omega$ is plotted in Fig. 4. From Eq. (5), we can conclude that the bandwidths for both the RCP and LCP cases are the same. The quantity $\Delta\omega$, like ω_0, first decreases then increases at the critical point $\rho = 1/2$. It is shown that the bandwidth increases with the increasing dielectric constant of background materials while decreases with the external magnetic field $\omega_{\rm ce}$. From the above discussions, we can conclude that one can control the behavior of negative refraction by using the proposed plasma metamaterials. Moreover, the properties of the negative refraction (i.e. the critical frequency and the bandwidth) is also tuned easily by suitable parameters, such as plasma density, plasma filling factor, dielectric constant of the background materials, and the external magnetic field. Compared with Ref. [28], it is clear that the external magnetic field is also a means of controlling the negative refraction in the plasma metamaterials. This is very easily understood as plasma can be controlled by changing the external magnetic field, while the parameters of plasma in turn determine the properties of the proposed plasma metamaterials.

In summary, we have demonstrated a uniaxial anisotropic plasma metamaterial that exhibits negative refraction and derived the necessary conditions for negative refraction in both the RCP and LCP cases. The results show that the parameters (i.e. the plasma filling factor, the dielectric constant of background materials and the external magnetic field) have significant effects on the critical frequency and bandwidth of negative refraction. Therefore, we can control the behavior of the negative refraction in plasma metamaterials by tuning the suitable parameters. However, as the losses are one main component that affects the properties of metamaterials, it would be of interest to deal with the losses in the plasma layer although we neglect the losses in the present study. Work in these areas is still necessary and in progress.

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